

# The HIGH SCHOOL ARITHMETIC



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# HIGH SCHOOL ARITHMETIC

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TORONTO

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## PREFACE

This Arithmetic has been prepared for the use of pupils in High and Continuation Schools and Collegiate Institutes.

It is largely a revision of the High School Arithmetic, which has been the authorized text for many years.

In addition to a re-arrangement of the contents of the first edition, the completion of the text has been effected by the introduction of such elementary work as would furnish the necessary review.

The treatment of Vulgar and Decimal Fractions introduces the pupil to a series of propositions of close mathematical reasoning, with the results of which he is already more or less familiar.

The formal treatment of Cube Root has been omitted, as being too cumbersome and intricate for any practical use that can be made of it by the average High School pupil.

In the Mensuration a number of experimental methods for finding the surfaces and volumes of the more difficult solids have been introduced.

Problems on the Metric System are included with those on Compound Quantities and on Mensuration.

A chapter on Miscellaneous Theorems and Applications has been inserted, containing much information of an interesting and practical nature.

## PREFACE

The Commercial Arithmetic has been made as complete and practical as possible. Full explanations of commercial terms and usages are given, clearly illustrated by concrete examples. The formal treatment of Annuities has been omitted, but the simpler cases, such as mortgages, etc., are included in Present Worth.

The Interest and Present Worth Tables will be found valuable in solving many of the problems on Compound Interest and Present Worth. They will also apprise pupils of the existence of such tables for facilitating the longer computations.

The General Problems in the first edition have been re-arranged and reduced in number, and many new ones have been added. They have been carefully graded in sets of 100 each, and should furnish an excellent review of the more important parts of arithmetic.

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# ARITHMETIC

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## Introduction

Whatever is capable of increase or diminution may be called a **Magnitude** or **Quantity**.

Thus, the length of a room, a sum of money, the weight of a ball, etc., are magnitudes or quantities.

If it is desired to measure or determine any quantity, it is impossible to do so except by selecting some known quantity of the same kind, and ascertaining their mutual relation.

If it is proposed to measure, say, the length of a room, we may make use of some known length, such as a foot. If this known length, or **Unit**, must be repeated 18 times to measure the length of the room, we say that the length of the room is the magnitude or quantity 18 feet, that the unit of measurement is 1 foot, and that the **Relative Value** or **Ratio** of the quantity to the unit is the **Number** 18.

A **Number**, then, is the ratio of one magnitude or quantity to another of the same kind which has been chosen as the unit. The number is obtained by determining, either by counting or measuring, how many times the unit must be repeated to produce the given quantity. The number which represents the measure of the quantity, depends upon the unit of measurement which is chosen.

Thus, the ratio of the quantity \$15 to the unit \$1 is the number 15, but if the unit chosen is \$3 the number would be 5, or if the unit is 1 cent, the number would be 1500.

**Arithmetic** is the science which treats of numbers and the art of computing by them.

The number which expresses the result of counting or measuring may be written in **Words** or **Symbols**.

**Numeration** is the system of expressing numbers in words. The ordinary system of numeration is called the **Decimal System**, because it is based on the number ten.

**Notation** is the system of expressing numbers by means of symbols which are called numerals, digits or figures.

**NOTE**—It is assumed that the student is familiar with the words and symbols used in the decimal system of numeration and notation.

### EXERCISE

1. In the number 256, what does the digit 2 represent? The digit 5? The digit 6?

2. In the number 304, what is the use of the 0? What difference would it make if it were omitted?

3. Read the number 1234 in two different ways.

4. What is the largest number expressed by two digits? By three digits?

5. What is the smallest number expressed by the digits 3, 5, 4? What is the largest?

6. Write down all the different numbers which can be formed using the digits 1, 2, 0.

7. In the quantity \$33333 which 3 represents the largest sum of money? Which the smallest?

8. What are convenient units of money which might be used to pay a debt of \$35? A debt of 75c.? A debt of 3c.?

9. What is the greatest number of six digits which begins with 5 and ends with 3? What is the least?

10. Write in digits all the numbers which do not differ from 1000 by more than 4.

11. In Great Britain a billion is a million millions. How many ciphers are there in a billion using that notation? How many in our system?

12. If the unit \$5 and the number 100 are used to denote a person's debts, how much does he owe?

13. If the units 5 cents and 2 yards of ribbon are equal in value, how many yards of ribbon can be bought for a dollar?

14. If a quarter of a mile is expressed by the number 220, what unit of measurement has been used?

15. Two quantities A and B are represented by the same number, but the unit for measuring A is a cubic foot and for B a cubic yard. How much larger is B than A? What number would represent B if A were the unit of measurement?

16. What units might be used to express the capacity of a bin that holds 100 bushels? How many of each of these units must be taken?

# The Simple Rules

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## ADDITION

**Addition** is the process of finding a single quantity which is equal to two or more quantities together.

The result of the addition is called the **Sum** of the quantities and the quantities which have been added are called **Addends**.

The operation of addition can be applied only to quantities which are of the same kind. Thus, the sum of 3 apples and 5 apples is 8 apples, but the sum of 3 apples and 5 oranges cannot be expressed either as a number of apples or as a number of oranges.

When we say that the sum of 4 and 5 is 9, no unit being expressed, it is to be understood that a unit is implied and that the unit is the same for the three numbers.

The sign of addition is  $+$ , and is read plus; thus,  $5+7$  is read 5 plus 7.

The sign of equality is  $=$ , and is read equals or equal; thus,  $3+6=9$  is read 3 plus 6 equals 9.

When numbers are to be added, it is evident that the order in which they are taken does not affect the sum.

Thus,  $6+4+5=6+5+4=5+6+4$ , etc.

## SUBTRACTION

**Subtraction** is the process of finding the part of a quantity which remains when a smaller quantity has been taken from it.

The quantity which is taken away is called the **Subtrahend**, the quantity from which it is taken is called the **Minuend**, and the part which remains is called the **Difference** or **Remainder**.

Since, to find the difference between 9 and 4 is the same as to find the number which, when added to 4, will make 9, subtraction may be looked upon as the **Inverse** of addition.

The sign of subtraction is  $-$ , and is read minus; thus,  $7-3$  is read 7 minus 3 and signifies that 3 is to be subtracted from 7.

$649-231=418$  is read 649 minus 231 equals 418, and means:

- (1) That 418 must be added to 231 to make 649,  
∴ the difference is 418.
- (2) That, if 231 be taken from 649, 418 is left,  
∴ the remainder is 418.

## MULTIPLICATION

**Multiplication** is the process of finding the sum of a given number of repetitions of the same quantity.

The quantity which is to be repeated or multiplied is called the **Multiplicand**.

The number which expresses how often the multiplicand is repeated is called the **Multiplier**.

The result of the multiplication is called the **Product**.

The multiplier and multiplicand are **Factors** of the product.

From the definition of the multiplier it is evident that it must be a number not a quantity.

The sign of multiplication is  $\times$ , and is read multiplied by or times.

Thus,  $\$4 \times 6 = \$24$  is read \$4 multiplied by 6 equals \$24.

When written  $6 \times \$4$ , it is read 6 times \$4.

In both cases the multiplicand is \$4, the multiplier is 6 and the product is \$24.

When the product of two numbers is multiplied by a third, the result is called the **Continued Product** of the numbers.

Thus the continued product of 2, 3 and 4 is 24. It is evident that the product is not affected by the order in which the numbers or factors are multiplied.

The continued product of a number with itself is called a power of the number. Thus,  $46 \times 46$  is the **Second Power** of 46 or the **Square** of 46, and is written  $46^2$ . The third power or cube is written  $46^3$  and so on. In  $46^3$  the 3 is called the **Exponent** or **Index** of the power.

## DIVISION

In **Multiplication** we are given two factors and are required to find their product.

In **Division** we are given the product and one of the factors, and are required to find the other factor. Thus, the product of 13 and 15 is found by multiplying to be 195.

If we are given the product 195 and one of the factors 13, the process by which the other factor 15 is obtained is called **Division**. The product is called the **Dividend**, the factor which is given, the **Divisor**, and the factor which is found, the **Quotient**.

The dividend is thus the product of the divisor and the quotient.

When the divisor is not contained an exact number of times in the dividend, the excess is called the **Remainder**.

When 46 is divided by 7, 6 is also called the quotient and the remainder is 4, which shows that  $46 = 7 \times 6 + 4$ , and, generally, the dividend = divisor  $\times$  quotient + remainder.

The sign of division is  $\div$  and is read divided by; thus,  $30 \div 5 = 6$  is read 30 divided by 5 equals 6.

### EXERCISE

1. Find the sum of all numbers between 6997 and 7009.
2. The gross earnings of a railway in 1908 were \$1,325,427.33. The net earnings were \$512,367.13. What were the expenses?
3. How many numbers are there between 150 and 500, both inclusive?
4. Find the sum of all numbers of three digits, which can be formed with the digits 7, 8, 9, without repeating any of the digits in any number.
5. Find the continued product of 123, 234, 345.
6. Which is the greater, the product of 2345 and 5678 or of 2543 and 5876, and by how much?
7. How many whole numbers are there with four digits?
8. A, B and C together have \$1000, A and B have \$761, B and C \$675. How much has each?

9. Divide 4325 into two numbers which differ by 215.

10. Supply the missing digits in the following subtractions:

$$\begin{array}{r} (1) \quad 4***4 \\ \quad *947* \\ \hline \quad 26205 \end{array}$$

$$\begin{array}{r} (2) \quad *84562 \\ \quad \quad 8*1*3 \\ \hline \quad 6*7*7* \end{array}$$

11. What is the least whole number above 2000 which is divisible by 29?

12. The expense of carpeting a room was \$45; but if the breadth had been 3 feet less, the expense would have been \$36. Find the breadth of the room.

13. The product of two numbers is 5720575 and one of them is 679. Find the other.

14. How many numbers less than 400 are divisible by 3? How many by 7? How many by both 3 and 7?

15. Find the average of 235, 123 and 326.

NOTE.—Find their sum and divide by three.

16. A drover bought 12 sheep at \$7.25 per head, 15 at \$6.15, and 24 at \$5.60. Find the total cost and the average cost per head.

17. The population of Winnipeg in 1901 was 42340 and in 1906 was 90150. What was the average yearly increase?

18. How often does the sum of 81459 and 54306 contain their difference?

19. What digit must be placed after 2354 so that the result may be exactly divisible by 17?

20. What three digits must be placed after 656459 to make the result exactly divisible by 1899?

21. What is the total number of digits required in numbering the pages of a book of 500 pages?

22. The population of Ontario in 1901 was 2,182,947, in 1891 it was 2,114,327. What was the average annual increase?

23. How much water must be mixed with 100 gallons of wine, worth \$2.50 a gallon, to make a mixture worth \$2 a gallon?

24. A Manitoba farmer sowed 85 acres of wheat, 67 of oats and 48 of barley. The wheat yielded 25, the oats 45 and the barley 35 bushels per acre. His expenses averaged \$2.20 an acre, what did he gain by selling the wheat at 64, the oats at 37 and the barley at 49 cents per bushel?

25. The Iliad contains 15683 lines and the Æneid 9892 lines. How many days will it take a boy to read both if he reads 75 lines a day?

26. In a recent election the number of votes polled by the candidates was as follows: A, 7,678,33; B, 6,407,340; C, 426,337; D, 248,881; E, 127,097. What was the total vote? What was A's majority? What was his plurality?

27. Supply the missing digits in the following multiplications:

$$\begin{array}{r} (1) \quad 823 \\ \quad ** \\ \hline \end{array}$$

$$\begin{array}{r} ***5 \\ *** \\ \hline ***** \end{array}$$

$$\begin{array}{r} (2) \quad *52* \\ \quad 3* \\ \hline \end{array}$$

$$\begin{array}{r} ***8 \\ 4**2 \\ \hline 4**** \end{array}$$

28. A farmer mixed 15 bushels of oats worth 40c. per bushel, with 5 bushels of corn worth 80c. per bushel. What is the mixture worth per bushel?

29. A produce merchant exchanged 48 bushels of oats worth 39c. a bushel, and 13 barrels of apples worth \$3.85 a barrel, for 200 pounds of butter worth 37c. a pound. How much should he pay to settle the account?

30. A share of railway stock costs \$78.50. How many shares may be bought for \$3061.50?

31. The area of the United Kingdom is 121,370 square miles. The population is 43,221,000. Find, to the nearest integer, the population per square mile.

32. The total expenditure for higher education in Canada in a recent year was \$1,040,260. The population was 5,201,300. What was the expenditure per capita?

33. A merchant's cash sales for a week were: Mon. \$34.75, Tues. \$42.34, Wed. \$31.14, Thurs. \$62.79, Fri. \$15.24, Sat. \$73.84. Find his average daily sales.

34. Supply the missing digits in the following divisions:

$$(1) \begin{array}{r} 8 \overline{) 3*2*} \\ *2*-1 \end{array}$$

$$(2) \begin{array}{r} 7 \overline{) 3*5*8} \\ *93*-2 \end{array}$$

35. If the dividend is 2468, the quotient 70, and the remainder 18, what is the divisor?

36. Multiply 42136 by 497, using only two partial products.

37. Multiply 3426 by 25 by first multiplying by 100 and then dividing by 4.

38. Multiply 32571 by 125 in a similar way to that in example 37.

39. Divide 479625 by 125 and also by 25 in a similar way.

40. Divide 325935 by 15 by short division by first dividing by 3 and then by 5.

41. Divide 421575 by 35 by short division.

42. Divide 3217 by 35 by short division, showing how you get the proper remainder.

43. Multiply 31276 by 9 by first multiplying by 10 and then subtracting 31276. Multiply also by 99 and 999 in a similar way.

# Measures and Multiples

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## GREATEST COMMON MEASURE

Any unit of length which will divide exactly into 18 inches is said to be a **Measure** or **Factor** of 18 inches.

Thus 6 inches is a measure or factor of 18 inches. 18 inches is called a **Multiple** of 6 inches.

Other factors or measures (omitting 1) of 18 are 2, 3, 9 and 18. Of these 2 and 3 are called **Prime Factors** because they have no other factors but themselves and unity.

Thus, the prime factors of 63 are 3 and 7; of 30 are 2, 3 and 5.

Since 6 will measure both 12 and 18, it is a **Common Measure** or **Factor** of 12 and 18.

Other common measures of 12 and 18 are 2 and 3, but the **Greatest common measure** is 6.

If all the prime factors of two or more numbers be written down, the greatest common measure (G. C. M.) may be obtained by inspection.

$$\text{Thus, } 45 = 3 \times 3 \times 5$$

$$105 = 3 \times 5 \times 7$$

$$135 = 3 \times 3 \times 3 \times 5$$

Here it is seen that 3 and 5, and therefore 15, are common measures of all the numbers, therefore the G. C. M. of 45, 105 and 135 is 15. If, however, the num-

bers cannot be readily factored, as say, 247 and 437, another method must be adopted which depends upon the following principle:

If 3 is a common factor of any two numbers, say, 15 and 21, then 3 is also a factor of  $15+21$ ,  $21-15$ ,  $4.21+5.15$ ,  $7.21-4.15$ , etc., for 3 will divide into any of these as here shown:

$$\begin{array}{r} 3)15+21 \\ \hline 5+7 \end{array}, \quad \begin{array}{r} 3)21-15 \\ \hline 7-5 \end{array}, \quad \begin{array}{r} 3)4.21+5.15 \\ \hline 4.7+5.5 \end{array}, \quad \begin{array}{r} 3)7.21-4.15 \\ \hline 7.7-4.5 \end{array}.$$

The reasoning here used does not depend upon the particular numbers, and is therefore general.

It follows then, that any common factor of two numbers is also a factor of the sum or difference of any multiples of those numbers.

To find the G. C. M. of 247 and 437, divide as shown in the margin, and 19 is the G. C. M. For  $190 = 437 - 247$ .

$$\begin{array}{r} 247)437(1 \\ \underline{247} \\ 190)247(1 \\ \underline{190} \end{array}$$

$\therefore$  every common factor of 437 and 247 is a factor of 190.

$$\begin{array}{r} 57)190(3 \\ \underline{171} \end{array}$$

$\therefore$  the G. C. M. of 437 and 247 is the G. C. M. of 190 and 247.

$$\begin{array}{r} 19)57(3 \\ \underline{57} \end{array}$$

But  $57 = 247 - 190$ .

$\therefore$  the G. C. M. of 190 and 247 is the G. C. M. of 57 and 190.

But  $19 = 190 - 3 \times 57$ .

$\therefore$  the G. C. M. of 57 and 190 is the G. C. M. of 19 and 57.

But  $57 = 3 \times 19$ .

$\therefore$  the G. C. M. of 19 and 57 is 19.

$\therefore$  the G. C. M. of 247 and 437 is 19.

To find the G. C. M. of more than two numbers, which cannot be readily factored, say of 442, 598 and 377, find the G. C. M. of any two of them by the preceding method.

Thus, the G. C. M. of 442 and 598 is 26.

Then the G. C. M. of the three numbers must be the G. C. M. of 26 and 377, which is 13.

### LEAST COMMON MULTIPLE

The number 48 is a multiple of both 6 and 8.

Other Common Multiples of 6 and 8 are 24, 72, 168, and so on.

The Least common multiple (L. C. M.) is 24.

It is evident that 24 could not be a multiple of 6 unless it contained all the factors of 6, that is, 2 and 3, nor could it be a multiple of 8 unless it contained all the factors of 8, that is,  $2 \times 2 \times 2$ .

The L. C. M. of two or more numbers is therefore the least number which contains all the prime factors of each of the given numbers.

Thus, to find the L. C. M. of 15, 25, 63.

$$15 = 3 \times 5$$

$$25 = 5 \times 5$$

$$63 = 3 \times 3 \times 7$$

The L. C. M. must contain the factors 3 and 5 to make it a multiple of 15, an additional factor 5 to make it a multiple of 25, and two additional factors, 3 and 7, to make it a multiple of 63.

$$\therefore \text{the L. C. M. is } 3 \times 5 \times 5 \times 3 \times 7 = 1575.$$

If the prime factors of the numbers cannot readily be found, then find the G. C. M. of them in pairs, and the prime factors may usually be written down without difficulty.

Thus, to find the L. C. M. of 442, 598 and 377.

The G. C. M. of 442 and 598 is 26.

The G. C. M. of 442 and 377 is 13.

$$\therefore 442 = 26 \times 17 = 2 \times 13 \times 17$$

$$598 = 26 \times 23 = 2 \times 13 \times 23$$

$$377 = 13 \times 29.$$

$$\therefore \text{the L. C. M.} = 2 \times 13 \times 17 \times 23 \times 29.$$

If the numbers are very large, there may be some difficulty in writing down the prime factors of the numbers, even when the G. C. M. of each pair is found, as in the preceding example. When this is the case the L. C. M. of two of the numbers may be found, and then the L. C. M. of this result and the third number, and so on if there are more than three numbers.

If no pair of the numbers have a common factor, the L. C. M. is evidently the product of all the numbers.

**The product of the G. C. M. and L. C. M. of two numbers is equal to the product of the numbers.**

Take any two numbers, say, 30 and 126.

$$30 = 2 \times 3 \times 5, \quad 126 = 2 \times 3 \times 3 \times 7.$$

Since the L. C. M. is the product of all the factors of the first number, and all the additional factors of the second (3 and 7), and the G. C. M. contains all the factors com-

mon to both (2 and 3), it follows that the product of the L. C. M. and G. C. M. will be the product of all the factors of both numbers, and is therefore equal to the product of the two numbers. The L. C. M. of two numbers may therefore be obtained by first finding their G. C. M., and dividing it into the product of the numbers. Thus, the G. C. M. of 325 and 481 is 13, therefore the L. C. M. is

$$\frac{325 \times 481}{13} = 12025.$$

### EXERCISE

1. Find, by factoring, the G. C. M. of 40 and 56; 230 and 506; 102 and 114; 210, 462 and 546.

2. Find, by dividing, the G. C. M. of 741 and 893; 4807 and 9545; 495, 891 and 1155.

3. Find, by factoring, the L. C. M. of 24, 30 and 36; 30, 32, 36, 40 and 48.

4. Find the L. C. M., by first finding the G. C. M., of 481 and 1665; 2257 and 3589.

5. Find the least number which is divisible by all numbers up to 16 inclusive.

6. Factorize, that is, express as the product of prime factors: (1) 30030, (2) 26208.

7. How many prime numbers are there less than 50?

8. What are the prime factors common to 12012 and 9072?

9. Find the least number into which 616 and 385 will each divide evenly.

10. Find the greatest number which will divide into 13956 and 14565 and leave a remainder 7 in each case.

11. What is the least number of acres that will admit of being divided into farms containing 150, 200 or 250 acres each?

12. Four bells toll at intervals of 3, 7, 12 and 14 seconds and begin to toll at the same instant. When will they next toll together?

13. A number greater than 300 and less than 400 is divisible by 2, 3, 4 and 5. What is the number?

14. Find the greatest number which on dividing 34100 leaves a remainder 63, and 307100 a remainder 93.

15. The L. C. M. of two numbers is 244188 and their G. C. M. is 84. If one of the numbers is 1428, find the other.

16. The L. C. M. of two numbers is 96. One of the numbers is 6. What may the other number be?

17. The L. C. M. of 68, 170 and another number is 2380, and the G. C. M. of the three numbers is 17. What may the other number be?

18. The sides of a rectangle are 465 and 682. If it is divided into squares, the side of each square being an integer and as large as possible, how many squares would there be?

19. How many rails will enclose a field 3143 feet long and 2471 feet wide, if the fence is straight and 8 rails high, and the longest that can be used?

20. A pound avoirdupois contains 7000 grains, and a pound Troy contains 5760 grains. Find the greatest weight which will measure each of them. Find also the least weight which each will measure.

# Fractions

---

The numbers which have so far been considered begin with 1, and go on increasing by 1, each number being greater by 1 than the number just before it. Such numbers are called **Whole Numbers** or **Integers**.

Besides these there are other numbers, some of them, such as *one-half*, *three-quarters*, being less than 1, and others such as *two and a half*, *three and a quarter*, being greater than one whole number and less than the next higher whole number. These numbers are called **Fractions**, those less than 1 being called **Proper Fractions**, and those greater than 1 **Improper Fractions**.

NOTE.—It will be seen hereafter that whole numbers may also be regarded as fractions, and as such are classed as improper fractions.

## PROPER FRACTIONS

If a unit be divided into two equal parts, each of these parts is called a *half*.

If into three equal parts, a *third*.

If into four equal parts, a *fourth*, and so on, the part in each case taking its name from the number of these parts required to make up the whole unit.

And we *speak* of any number of these parts in the same way that we do of a number of feet, pounds or dollars; but the notation employed when we come to *write* them is different.

Thus for three dollars we write \$3; for four pounds we write 4 lb.; where the character or symbol denoting the *kind* of thing under consideration is placed to the right or left (and sometimes directly above) that denoting the *number* of things; but for three-fourths we write  $\frac{3}{4}$ ; the 4 in this expression, which tells the kind of thing under consideration, namely, *fourths*, being placed directly below the 3, which tells the number of fourths.

This symbol  $\frac{3}{4}$  is called a **Fraction**, and indicates that a unit of some kind has been divided into 4 equal parts, and that 3 of these parts are under consideration.

The 4 which tells the kind or **Denomination** of the parts (and therefore also the number of parts into which the unit has been divided), is called the **Denominator**. The 3, which indicates the number of parts taken (*i.e.*, *enumerates* them) is called the **Numerator**. The numerator and denominator are called the **Terms** of the fraction.

If the numerator of a fraction be a smaller number than the denominator, it is evident that the number of parts taken is less than the number of parts into which the unit was divided, and therefore that the fraction denotes a number less than unity or 1. Such fractions are accordingly called **Proper** fractions.

## IMPROPER FRACTIONS

Four fourths or five fifths of any unit are evidently equal to the whole of that unit, and therefore every such fraction as  $\frac{4}{4}$ ,  $\frac{5}{5}$  or  $\frac{6}{6}$  is equal to 1.

The word fraction (from *fractus*, broken) strictly means a part broken off, and consequently denotes something less than the whole. Such expressions therefore as  $\frac{4}{4}$ ,  $\frac{5}{5}$  or  $\frac{6}{6}$ , not indicating a *part* of a unit, are called **Improper fractions**.

Now, suppose two or more units to be divided each into five equal parts, and that any number of these parts is taken, then such fractions as  $\frac{7}{5}$ ,  $\frac{13}{5}$ , etc., may be obtained, where the numerator is greater than the denominator. These are also called **Improper fractions**.

## MIXED NUMBERS

Take the improper fraction  $1\frac{3}{5}$ .

Since 5 fifths are equal to 1,

$\therefore$  10 fifths are equal to 2,

$\therefore$  13 fifths are equal to 2 and 3 fifths.

It therefore follows that every improper fraction is equivalent to a whole number, or else to a whole number and a proper fraction, and that the whole number can be obtained by finding how often the denominator of the fraction is contained in the numerator, and also that the remainder, if any, after dividing, will be the numerator of the proper fraction.

Such numbers are expressed by writing the whole number first with the fraction close after it; thus, *two and three-fifths* would be written  $2\frac{3}{5}$ . Such numbers are called **Mixed Numbers**. Thus every improper fraction may be expressed either as a whole or as a mixed number.

### EXERCISE

Express the following improper fractions as whole or mixed numbers:

- |                          |                           |                           |                            |
|--------------------------|---------------------------|---------------------------|----------------------------|
| 1. $\frac{13}{4}$ .      | 2. $\frac{27}{3}$ .       | 3. $\frac{47}{8}$ .       | 4. $\frac{62}{7}$ .        |
| 5. $\frac{327}{12}$ .    | 6. $\frac{99}{11}$ .      | 7. $\frac{768}{24}$ .     | 8. $\frac{3482}{43}$ .     |
| 9. $\frac{43875}{963}$ . | 10. $\frac{10000}{909}$ . | 11. $\frac{10101}{101}$ . | 12. $\frac{101101}{101}$ . |

Conversely, every mixed number may be expressed as an improper fraction.

For, consider the number  $3\frac{4}{5}$ .

Since 1 is equal to 5 fifths,

$\therefore$  3 is equal to 15 fifths,

$\therefore 3\frac{4}{5}$  is equal to 15 fifths and 4 fifths,

$\therefore 3\frac{4}{5} = 19$  fifths, or  $\frac{19}{5}$ .

### EXERCISE

Reduce the following mixed numbers to improper fractions:

- |                        |                          |                           |                            |
|------------------------|--------------------------|---------------------------|----------------------------|
| 1. $3\frac{1}{2}$ .    | 2. $5\frac{1}{4}$ .      | 3. $6\frac{7}{8}$ .       | 4. $11\frac{1}{2}$ .       |
| 5. $17\frac{5}{7}$ .   | 6. $57\frac{9}{10}$ .    | 7. $14\frac{1}{5}$ .      | 8. $23\frac{8}{9}$ .       |
| 9. $3476\frac{1}{9}$ . | 10. $969\frac{19}{23}$ . | 11. $2\frac{2345}{789}$ . | 12. $576\frac{576}{777}$ . |

### COMPOUND FRACTIONS

It is evident that if 3 fourths be repeated 7 times the result will be 21 fourths; 6 sevenths repeated 5 times the result will be 30 sevenths; and so on.

That is,  $\frac{3}{4} \times 7 = \frac{21}{4}$ ;  $\frac{6}{7} \times 5 = \frac{30}{7}$ .

## EXERCISE

Multiply:

1.  $\frac{6}{7}$  by 4, 6, 8, 10 and 16.

2.  $\frac{17}{19}$  by 9, 13, 23 and 47.

3.  $\frac{100}{157}$  by 49, 93 and 256.

If any unit be divided into 4 equal parts and each of these fourths be divided into 5 equal parts, it is evident that the unit will be divided into 20 equal parts. From this it follows that the fifth part of a fourth is a twentieth; or, as it is written:

$$\frac{1}{5} \text{ of } \frac{1}{4} = \frac{1}{20}$$

and since 1 fourth is equal to 5 twentieths,

$\therefore$  3 fourths are equal to 15 twentieths,

$\therefore$  1 fifth of 3 fourths is equal to 1 fifth of 15 twentieths, and therefore equal to 3 twentieths; or, as it is written:

$$\frac{1}{5} \text{ of } \frac{3}{4} = \frac{3}{20}.$$

It is clear also that 2 fifths of 3 fourths is twice as much as 1 fifth of 3 fourths, and is therefore equal to 6 twentieths, that is:

$$\frac{2}{5} \text{ of } \frac{3}{4} = \frac{6}{20}.$$

From this it appears that a fraction of a fraction may be reduced to a simple fraction by multiplying the two numerators together for a new numerator, and the two denominators together for a new denominator.

$$\text{Similarly } \frac{3}{7} \text{ of } \frac{2}{5} \text{ of } \frac{3}{4} = \frac{3}{7} \text{ of } \frac{6}{20} = \frac{18}{140} = \frac{3 \times 2 \times 3}{7 \times 5 \times 4}.$$

Such expressions as  $\frac{2}{5}$  of  $\frac{3}{4}$  and  $\frac{3}{7}$  of  $\frac{2}{5}$  of  $\frac{3}{4}$  are called **Compound Fractions**.

## EXERCISE

Reduce the following compound fractions to simple ones:

1.  $\frac{1}{2}$  of  $\frac{1}{3}$ .
2.  $\frac{6}{7}$  of  $\frac{8}{11}$ .
3.  $\frac{2}{3}$  of  $\frac{4}{5}$  of  $\frac{7}{9}$ .
4.  $\frac{17}{100}$  of  $\frac{11}{200}$  of  $\frac{9}{1000}$ .
5.  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $\frac{1}{4}$  of  $\frac{1}{5}$  of  $\frac{1}{6}$ .
6.  $\frac{10}{19}$  of  $\frac{100}{17}$  of  $\frac{13}{3}$ .
7.  $\frac{2}{3}$  of  $5\frac{1}{3} = \frac{2}{3}$  of  $\frac{16}{3}$ .
8.  $\frac{1}{12}$  of  $19\frac{3}{7}$  of  $100\frac{9}{10}$ .
9.  $\frac{8}{9}$  of  $7\frac{1}{3}$  of  $4\frac{1}{3}$ .
10.  $\frac{1}{3}$  of  $1\frac{999}{1000}$  of  $\frac{1}{999}$ .

Since one-fifth of  $\frac{3}{4}$  is  $\frac{3}{20}$ , and since one-fifth of anything is the result obtained after dividing it by 5, it follows that  $\frac{3}{4}$  on being divided by 5 gives  $\frac{3}{20}$  for quotient, or

$$\frac{3}{4} \div 5 = \frac{3}{20}.$$

## EXERCISE

Divide:

1.  $\frac{5}{7}$  by 2, 3, 4 and 8.
2.  $\frac{7}{18}$  by 8 and by 10.
3.  $\frac{148}{75}$  by 125.

If the fraction  $\frac{3}{4}$  be multiplied by 5 the result will be  $\frac{15}{4}$ , and if  $\frac{15}{4}$  be divided by 5 the result will be  $\frac{15}{20}$ ; but it is evident that these operations must give a final result equal to the original fraction.

It follows, therefore, that  $\frac{15}{20}$  and  $\frac{3}{4}$  must be equal to each other.

Hence, if both terms of a fraction be multiplied by the same number, the value of the fraction remains the same.

Thus, to reduce  $\frac{5}{7}$  to a fraction whose denominator is 56, multiply both terms of the fraction by 8 (the number of times 7 is contained in 56), and the result is  $\frac{40}{56}$ .

## EXERCISE

1. Change  $\frac{3}{4}$  to an equivalent fraction whose denominator is 12.  
20. 48. 100.

2. Reduce  $\frac{5}{7}$  and  $\frac{3}{4}$  to fractions, each of which will have 28 for denominator.

3. Reduce  $\frac{7}{9}$  and  $\frac{6}{7}$  to fractions which will have the same denominator.

4. Reduce  $\frac{2}{3}$ ,  $\frac{3}{4}$  and  $\frac{4}{5}$  to fractions which will have 60 for denominator.

5. Reduce  $\frac{5}{6}$ ,  $\frac{7}{8}$  to fractions having 24 for denominator.

6. Reduce  $\frac{5}{9}$ ,  $\frac{1}{6}$ ,  $\frac{2}{3}$  to fractions having 18 for denominator.

7. Reduce  $\frac{5}{6}$ ,  $\frac{7}{9}$ ,  $\frac{1}{12}$ ,  $\frac{1}{2}$  and  $\frac{5}{18}$  to equivalent fractions which will have the same denominator.

Also, if both terms of  $\frac{15}{20}$  be divided by 5 the result is  $\frac{3}{4}$ , which has been shown to be equal to  $\frac{15}{20}$ .

Hence, if both terms of a fraction be divided by the same number, the value of the fraction remains the same.

$$\text{Thus, } \frac{30}{42} = \frac{15}{21} = \frac{5}{7}.$$

When the terms of a fraction have been divided by all numbers which will divide both of them, the fraction is said to be reduced to its **Lowest Terms**.

$$\text{Thus, } \frac{315}{728} \text{ (dividing by 5)} = \frac{63}{145.6} \text{ (dividing by 3)} = \frac{21}{48.5} = \frac{3}{7}.$$

## EXERCISE

Reduce the following fractions to their lowest terms:

- |                        |                         |                          |                         |
|------------------------|-------------------------|--------------------------|-------------------------|
| 1. $\frac{3}{9}$ .     | 2. $\frac{15}{20}$ .    | 3. $\frac{21}{28}$ .     | 4. $\frac{18}{27}$ .    |
| 5. $\frac{45}{75}$ .   | 6. $\frac{28}{98}$ .    | 7. $\frac{126}{189}$ .   | 8. $\frac{147}{198}$ .  |
| 9. $\frac{856}{768}$ . | 10. $\frac{243}{729}$ . | 11. $\frac{625}{1000}$ . | 12. $\frac{294}{378}$ . |

NOTE.—When common divisors can no longer be determined by inspection, the method of finding the G. C. M. of two numbers must be used.

13.  $\frac{188}{171}$ .

14.  $\frac{187}{221}$ .

15.  $\frac{63427}{77121}$ .

16.  $\frac{47957}{54549}$ .

In reducing compound fractions to simple ones the common factors should be cancelled before multiplying. Thus, in the following example:

Reduce  $\frac{8}{9}$  of  $\frac{15}{24}$  of  $\frac{3}{10}$ .

$$\text{Result} = \frac{8 \times 15 \times 3}{9 \times 24 \times 10}$$

Here it is seen that 8 occurs in the numerator, and is also contained in the denominator as a factor of 24; and the same is true of 3; cancelling these there is left

$$\frac{15}{9 \times 10}$$

Here again 3 occurs as a factor of 9 and also as a factor of 15, while 5 occurs as a factor of 15 and also as a factor of 10. Cancelling these common factors there is left

$$\frac{1}{3 \times 2} = \frac{1}{6}$$

as the final result.

### EXERCISE

Reduce to simple fractions in their lowest terms:

1.  $\frac{1}{3}$  of  $\frac{3}{4}$  of  $\frac{4}{5}$ .      2.  $\frac{2}{3}$  of  $\frac{6}{7}$  of  $\frac{14}{16}$ .      3.  $\frac{7}{9}$  of  $\frac{81}{147}$ .

4.  $\frac{12}{13}$  of  $\frac{65}{19}$  of  $\frac{57}{180}$ .      5.  $\frac{1}{11}$  of  $\frac{3}{5}$  of  $\frac{3}{4}$  of  $3\frac{2}{3}$ .

6.  $\frac{5}{2}$  of  $\frac{16}{7}$  of  $\frac{7}{18}$  of  $\frac{11}{10}$  of  $\frac{27}{14}$ .      7.  $\frac{1}{3}$  of  $\frac{9}{8}$  of  $2\frac{1}{3}$  of  $15\frac{2}{7}$ .

8.  $\frac{5}{7}$  of  $\frac{33}{49}$  of  $\frac{5}{4}$  of  $6\frac{1}{10}$  of  $\frac{14}{13}$  of  $2\frac{4}{11}$ .

9.  $\frac{2247}{1017}$  of  $\frac{774}{615}$  of  $\frac{1017}{903}$  of  $\frac{565}{1928}$ .

10.  $\frac{13}{17}$  of  $\frac{19}{23}$  of  $\frac{93}{155}$  of  $\frac{49}{133}$  of  $1\frac{8}{28}$  of  $1\frac{10}{105}$  of  $1\frac{7}{135}$  of  $1\frac{11}{58}$ .

To show that  $\frac{1}{5}$  of  $2 = \frac{2}{5}$ .

If each of two units be divided into 5 equal parts, each of these parts will be one-fifth, and there will be 10 of

them. If these 10 fifths be separated into groups of 2 each there will be 5 of these groups. Hence 2 has been divided into 5 equal parts, and each part consists of 2 fifths; that is

$$\frac{1}{5} \text{ of } 2 = \frac{2}{5}.$$

Also, since the fifth part of any number is obtained by dividing that number by 5, it follows that

$$\frac{2}{5} = \frac{1}{5} \text{ of } 2 = 2 \div 5.$$

Therefore every fraction expresses the quotient of the numerator by the denominator.

It will also follow that every whole number may be expressed as a fraction having 1 for its denominator; thus,

$$5 = \frac{5}{1}, \text{ for } \frac{5}{1} = 5 \div 1 = 5.$$

Therefore also a whole number may be expressed as a fraction having any proposed denominator. Thus, to express 7 as a fraction whose denominator is 5,

$$7 = \frac{7}{1} = \frac{35}{5}.$$

### EXERCISE

1. Express 9 as a fraction whose denominator is 6, 7, 10, 100.
2. Express 39 as a fraction whose denominator is 14, 19, 23.

## TO REDUCE FRACTIONS TO OTHERS HAVING A COMMON DENOMINATOR

Reduce the fractions  $\frac{2}{3}$ ,  $\frac{3}{5}$  and  $\frac{2}{7}$ , to other fractions having a common denominator.

This denominator must contain 3, 5 and 7 as factors,  
 $\therefore$  it must be a multiple of 3, 5 and 7, such as 105,  
 210, 315, etc.

$\therefore$  the required fractions would be

$$\begin{array}{l} \frac{70}{105}, \frac{63}{105}, \frac{30}{105}, \\ \text{or } \frac{140}{210}, \frac{126}{210}, \frac{60}{210}, \\ \text{or } \frac{210}{315}, \frac{189}{315}, \frac{90}{315}, \end{array}$$

and so on.

In reducing fractions to a common denominator, however, it is most convenient to select the least denominator that will contain all the given denominators, that is, to take the least common multiple of all the denominators.

To reduce

$$\frac{5}{8}, \frac{1}{6}, \frac{7}{12}, \frac{5}{9},$$

to fractions having the least common denominator.

The L. C. M. of 8, 6, 12, and 9 is 72.

The first denominator 8 is contained in 72, the L. C. M., 9 times, therefore the first fraction  $\frac{5}{8}$  becomes

$$\frac{5 \times 9}{8 \times 9} = \frac{45}{72}$$

Similarly, the second,  $\frac{1}{6}$  becomes  $\frac{12}{72}$

the third  $\frac{7}{12}$       “       $\frac{42}{72}$

and the fourth  $\frac{5}{9}$       “       $\frac{40}{72}$

The resulting fractions are, therefore,

$$\frac{45}{72}, \frac{12}{72}, \frac{42}{72}, \text{ and } \frac{40}{72}$$

## EXERCISE

Reduce the following fractions to equivalent ones with the least common denominator:

NOTE.—If the fractions are not in their lowest terms, they should first be reduced to their lowest terms.

1.  $\frac{7}{9}$ ,  $\frac{8}{15}$ ,  $\frac{1}{6}$ .
2.  $\frac{2}{3}$ ,  $\frac{3}{4}$ ,  $\frac{4}{5}$ ,  $\frac{5}{6}$ .
3.  $\frac{14}{15}$ ,  $\frac{7}{18}$ ,  $\frac{23}{24}$ ,  $\frac{19}{21}$ .
4.  $\frac{1}{10}$ ,  $\frac{1}{100}$ ,  $\frac{1}{1000}$ ,  $\frac{1}{10000}$ .
5.  $\frac{7}{39}$ ,  $\frac{16}{21}$ ,  $\frac{29}{91}$ .
6.  $\frac{5}{7}$ ,  $\frac{30}{119}$ ,  $\frac{1}{51}$ ,  $\frac{2}{3}$ ,  $\frac{1}{21}$ .
7.  $\frac{7}{28}$ ,  $\frac{5}{46}$ ,  $\frac{19}{24}$ ,  $\frac{13}{65}$ ,  $\frac{68}{69}$ .
8.  $\frac{1}{2}$  of  $\frac{3}{4}$ ,  $6\frac{7}{8}$ ,  $\frac{5}{9}$  of  $7\frac{1}{3}$ .
9.  $\frac{7}{18}$  of  $\frac{4}{5}$ ,  $11\frac{1}{18}$ ,  $\frac{1}{2}$  of  $\frac{18}{7}$  of  $3\frac{8}{9}$ .
10.  $\frac{1}{3}$  of  $\frac{1}{5}$  of  $\frac{1}{7}$ ,  $\frac{1}{5}$  of  $\frac{1}{7}$  of  $\frac{1}{11}$ ,  $\frac{1}{3}$  of  $\frac{1}{11}$  of  $\frac{1}{18}$ .

## TO COMPARE FRACTIONS IN MAGNITUDE

Determine which is the greater of the two fractions,  $\frac{4}{5}$  and  $\frac{5}{7}$ .

This cannot readily be done, as they have different denominators, for it cannot be said at once whether 4 out of 5 equal parts are greater or less than 5 out of 7 equal parts. But if they are changed into their equivalent fractions with a common denominator, they become respectively:

$$\frac{28}{35} \text{ and } \frac{25}{35}.$$

and since 28 out of 35 equal parts are greater than 25 of these parts,

$\therefore \frac{4}{5}$  is greater than  $\frac{5}{7}$ .

## EXERCISE

1. Which is the greater,  $\frac{6}{7}$  or  $\frac{7}{8}$ ?
2. Find the least of the fractions  $\frac{5}{6}$ ,  $\frac{7}{9}$  and  $\frac{14}{17}$ .

3. Arrange  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$  in order of magnitude, beginning with the greatest.
4. Arrange  $\frac{2}{8}$ ,  $\frac{7}{12}$ ,  $\frac{17}{24}$  in order.
5. Arrange  $\frac{1}{8}$  of  $\frac{3}{5}$ ,  $\frac{1}{12}$  and  $\frac{4}{7}$  of 2.
6. Arrange  $\frac{7}{12}$ ,  $\frac{11}{21}$ ,  $\frac{5}{18}$  and  $\frac{29}{60}$ .
7. Arrange  $\frac{16}{7}$ ,  $2\frac{3}{8}$ ,  $\frac{22}{9}$ , and  $2\frac{5}{11}$ .
8. Arrange  $\frac{3}{8}$  of  $\frac{5}{7}$  of 4,  $\frac{2}{7}$  of  $\frac{3}{11}$  of 7,  $\frac{1}{5}$  of  $\frac{1}{3}$  of  $5\frac{2}{5}$  and  $\frac{18}{27}$ .
9. Arrange  $\frac{18}{19}$ ,  $\frac{19}{20}$ ,  $\frac{22}{23}$  and  $\frac{23}{24}$ .

Take the **Proper Fraction**  $\frac{3}{5}$  and add 2 to each of its terms, the resulting fraction  $\frac{5}{7}$  is greater than  $\frac{3}{5}$ .

For  $\frac{3}{5}$  is  $\frac{2}{5}$  less than unity.

And  $\frac{5}{7}$  is  $\frac{2}{7}$  less than unity.

But  $\frac{2}{7}$  is evidently less than  $\frac{2}{5}$ .

Therefore  $\frac{5}{7}$  is greater than  $\frac{3}{5}$ .

Take the **Improper Fraction**  $\frac{11}{7}$  and add 3 to each of its terms, the resulting fraction  $\frac{14}{10}$  is less than  $\frac{11}{7}$ .

For  $\frac{11}{7}$  is  $\frac{4}{7}$  more than unity.

And  $\frac{14}{10}$  is  $\frac{4}{10}$  more than unity.

But  $\frac{4}{7}$  is evidently greater than  $\frac{4}{10}$ .

Therefore  $\frac{14}{10}$  is less than  $\frac{11}{7}$ .

In a similar way it will be seen that any fraction becomes more nearly equal to unity, by adding the same number to both of its terms; that is, if the fraction is a proper fraction it is increased, and if an improper fraction it is decreased.

What is the conclusion when the same number is subtracted from both terms of a fraction?

## EXERCISE

Arrange in ascending order of magnitude the fractions in the following examples:

$$1. \frac{7}{8}, \frac{6}{7}. \quad 2. \frac{16}{17}, \frac{18}{19}. \quad 3. \frac{21}{19}, \frac{17}{15}, \frac{25}{23}. \quad 4. \frac{7}{17}, \frac{16}{25}, \frac{12}{21}.$$

$$5. \frac{6}{7}, \frac{11}{13}, (\text{change } \frac{6}{7} \text{ to } \frac{12}{14}). \quad 6. \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}.$$

7.  $\frac{3}{97}, \frac{5}{161}, \frac{11}{353}, \frac{7}{225}$ . In this example the fractions may be written:

$$\frac{1}{32\frac{1}{3}}, \frac{1}{32\frac{1}{6}}, \frac{1}{32\frac{1}{11}}, \frac{1}{32\frac{1}{7}}.$$

$$8. \frac{3}{19}, \frac{5}{31}, \frac{7}{44}.$$

## ADDITION OF FRACTIONS

If 1 dollar, 2 dollars and 3 dollars be added together their sum will be 6 dollars, and so if 1 seventh, 2 sevenths and 3 sevenths be added together their sum will be 6 sevenths; or as it is written,

$$\frac{1}{7} + \frac{2}{7} + \frac{3}{7} = \frac{6}{7}.$$

It therefore follows that if fractions have the same denominator their sum is a fraction whose numerator is the sum of their numerators, and whose denominator is the same as their denominator.

If the fractions to be added have not the same denominator, as

$$\frac{2}{3}, \frac{3}{4}, \frac{4}{5},$$

they may be reduced to equivalent fractions having their least common denominator, as

$$\frac{40}{60}, \frac{45}{60}, \frac{48}{60},$$

and their sum can then be found by the method already given, thus:

$$\frac{40}{60} + \frac{45}{60} + \frac{48}{60} = \frac{133}{60} = 2\frac{13}{60}.$$

## EXERCISE

Find the sum of:

1.  $\frac{3}{5}, \frac{2}{5}, \frac{4}{5}$ .
2.  $\frac{3}{18}, \frac{7}{18}, \frac{2}{18}$ .
3.  $\frac{6}{18}, \frac{10}{18}, \frac{6}{18}$ .
4.  $\frac{3}{5}, \frac{5}{7}$ .
5.  $\frac{5}{6}, \frac{7}{8}$ .
6.  $\frac{17}{18}, \frac{19}{24}$ .
7.  $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ .
8.  $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$ .
9.  $\frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}$ .
10.  $\frac{1}{9}, \frac{1}{19}, \frac{1}{29}, \frac{1}{39}$ .
11.  $3\frac{1}{2}, 4\frac{1}{3}, 5\frac{1}{4}$ , or 3, 4, 5,  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ .
12.  $6\frac{1}{2}, 7\frac{1}{4}$ .
13.  $17\frac{3}{8}, \frac{47}{16}, \frac{13}{7}, \frac{5}{9}$ .

Find the value of:

14.  $\frac{2}{7} + 5\frac{2}{11} + 1\frac{1}{3}$  of  $2\frac{1}{2} + 6\frac{1}{4}$ .
15.  $\frac{2}{7}$  of  $\frac{5}{14} + \frac{4}{5}$  of  $\frac{1}{10} + \frac{3}{5}$  of  $(\frac{1}{2} + \frac{1}{14}) + \frac{3}{70}$  of  $(\frac{2}{7} + \frac{4}{5})$ .

First add the fractions within the brackets, then simplify the compound fractions.

## SUBTRACTION OF FRACTIONS

If from a group of 7 apples, 4 apples be taken, there will be 3 apples left, and so if from 7 ninths, 4 ninths be taken, there will be 3 ninths left, or as it is written,

$$\frac{7}{9} - \frac{4}{9} = \frac{3}{9}$$

It therefore follows that if two fractions have the same denominator their difference is a fraction whose numerator is the difference of their numerators, and whose denominator is the same as their denominator.

Find the difference between

$$\frac{5}{7} \text{ and } \frac{3}{8}.$$

It cannot be found in their present form, but if they are reduced to their equivalent fractions

$$\frac{40}{56} \text{ and } \frac{21}{56}$$

it follows that their difference is  $\frac{19}{56}$ .

## EXERCISE

Find the difference between:

1.  $\frac{4}{5}$  and  $\frac{1}{5}$ .
2.  $\frac{13}{17}$  and  $\frac{11}{17}$ .
3.  $\frac{7}{8}$  and  $\frac{3}{4}$ .
4.  $\frac{1}{2}$  and  $\frac{1}{3}$ .
5.  $\frac{1}{11}$  and  $\frac{1}{13}$ .
6.  $\frac{11}{24}$  and  $\frac{15}{32}$ .
7.  $\frac{98}{99}$  and  $\frac{65}{66}$ .
8.  $37\frac{8}{9}$  and  $14\frac{2}{7}$ .
9.  $\frac{1}{4}$  of  $\frac{1}{5}$  and  $\frac{3}{7}$  of  $\frac{2}{9}$ .
10. The sum of  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{4}$  and the sum of  $\frac{6}{7}$  and  $\frac{4}{5}$ .
11. The sum of  $\frac{1}{18}$  and  $\frac{1}{19}$  and their difference.

Find the value of:

12.  $\frac{7}{8} - \frac{6}{7}$ .
13.  $\frac{28}{29} - \frac{27}{28}$ .
14.  $\frac{3}{7}$  of  $\frac{14}{21} - \frac{1}{7}$ .
15.  $\frac{6}{7} + \frac{8}{10} - \frac{19}{20}$ .
16.  $5\frac{1}{3} - 3\frac{1}{5} = (5-3) + (\frac{1}{3} - \frac{1}{5})$ .
17.  $17\frac{3}{11} - 15\frac{5}{9} = 2\frac{3}{11} - \frac{5}{9}$ .
18.  $\frac{7}{8} - \frac{2}{5} + \frac{15}{16} - \frac{1}{4} - \frac{1}{2}$  (by three different methods).

## MULTIPLICATION OF FRACTIONS

If 8 be multiplied by 12 the product is 96.

If 8 be multiplied by 6, which is  $\frac{1}{2}$  of 12, the product is 48, which is  $\frac{1}{2}$  of 96.

If 8 be multiplied by  $\frac{1}{3}$  of 12, the product is  $\frac{1}{3}$  of 96.

Similarly we will define that if 8 be multiplied by  $\frac{1}{6}$  of 12, the product is  $\frac{1}{6}$  of 96.

Or that  $8 \times \frac{1}{6}$  of 12 =  $\frac{1}{6}$  of 96.

But  $\frac{1}{6}$  of 12 =  $\frac{12}{6}$  and  $\frac{1}{6}$  of 96 =  $\frac{96}{6}$ ,

$\therefore 8 \times \frac{12}{6} = \frac{96}{6}$ .

To multiply 8 by  $\frac{12}{6}$  then, we first multiply 8 by 12, and then take  $\frac{1}{6}$  of the product.

Similarly to multiply  $\frac{8}{4}$  by  $\frac{12}{6}$  we first multiply  $\frac{8}{4}$  by 12, which gives  $\frac{24}{4}$ .

Then  $\frac{1}{7}$  of  $\frac{8}{7} = \frac{8}{35}$ .

Therefore  $\frac{8}{7} \times \frac{12}{5} = \frac{8 \times 12}{7 \times 5} = \frac{96}{35}$ .

Similarly  $\frac{2}{3} \times \frac{4}{5} \times \frac{2}{7} = \frac{2 \times 4}{3 \times 5} \times \frac{2}{7} = \frac{2 \times 4 \times 2}{3 \times 5 \times 7} = \frac{16}{105}$ .

Before multiplying the numerators together, or the denominators, all factors common to a numerator and a denominator should be cancelled.

### EXERCISE

Find the product of:

1.  $\frac{2}{3}$  and  $\frac{4}{9}$ .
2.  $\frac{5}{7}$  and  $\frac{2}{3}$ .
3.  $\frac{2}{5}$ ,  $\frac{1}{3}$  and  $\frac{4}{7}$ .
4.  $\frac{7}{10}$ ,  $\frac{7}{9}$  and  $\frac{7}{8}$ .
5.  $\frac{3}{4}$ ,  $\frac{4}{5}$ ,  $\frac{5}{6}$  and  $\frac{6}{7}$ .
6.  $4\frac{1}{3}$  and  $\frac{3}{13}$ .
7.  $\frac{3}{4}$  of  $\frac{15}{17}$  and  $\frac{5}{9}$  of 6.
8.  $2\frac{1}{2}$ ,  $3\frac{1}{3}$ ,  $3\frac{3}{4}$  and  $5\frac{1}{5}$ .
9.  $\frac{1}{2} + \frac{1}{3}$  and  $\frac{1}{3} + \frac{1}{4}$ .
10.  $\frac{1}{4} + \frac{1}{5}$  and  $\frac{1}{4} - \frac{1}{5}$ .

Find the value of:

11.  $\frac{2}{3} \times \frac{3}{4} \times 4\frac{1}{2} \times \frac{8}{27}$ .
12.  $\frac{3}{4} \times \frac{6}{7} - \frac{1}{2}$ .
13.  $\frac{3}{4} \times (\frac{6}{7} - \frac{1}{2})$ .
14.  $(\frac{3}{5} \text{ of } 7\frac{1}{2} - \frac{8}{17}) \times \frac{9}{11}$ .
15.  $\frac{5}{6} \times \frac{9}{10} - \frac{3}{7} \text{ of } \frac{7}{9} + \frac{6}{7} \times (\frac{2}{3} - \frac{1}{2})$ .

In 15 and similar examples the order of reduction is as follows.

1. Find the value of the fractions in brackets.
2. Simplify the compound fraction  $\frac{3}{7}$  of  $\frac{7}{9}$ .
3. Perform the multiplications indicated.
4. Perform the additions and subtractions indicated in the order in which they occur.

## DIVISION OF FRACTIONS

When integers are used we say that

$$\begin{aligned} &\text{as } 7 \times 6 = 42, \\ &\text{then } 42 \div 6 = 7. \end{aligned}$$

Similarly when fractions are used we will define that

$$\begin{aligned} &\text{as } \frac{8}{7} \times \frac{12}{5} = \frac{96}{35}, \\ &\text{then } \frac{96}{35} \div \frac{12}{5} = \frac{8}{7}. \\ &\text{But } \frac{96}{35} \times \frac{5}{12} = \frac{8}{7}, \\ &\therefore \frac{96}{35} \div \frac{12}{5} = \frac{96}{35} \times \frac{5}{12}. \end{aligned}$$

## EXERCISE

Divide:

1.  $\frac{2}{5}$  by  $\frac{1}{2}$ .
2.  $\frac{5}{9}$  by  $\frac{5}{18}$ .
3.  $\frac{48}{49}$  by  $\frac{13}{14}$ .
4.  $6\frac{7}{9}$  by  $8\frac{5}{7}$ .
5.  $\frac{187}{209}$  by  $\frac{204}{216}$ .
6.  $\frac{19}{20}$  by  $\frac{4}{5}$  of  $6\frac{1}{4}$ .
7.  $\frac{9}{10} \times \frac{5}{7}$  of 14 by 99.
8.  $(\frac{6}{7} + \frac{4}{5} - \frac{5}{7})$  by  $\frac{6}{7}$  of  $(\frac{9}{10} - \frac{8}{9})$ .

Find the value of:

9.  $\frac{5}{8} \div \frac{8}{10}$ .
10.  $\frac{7}{8}$  of  $4\frac{1}{2} \div \frac{7}{16}$ .
11.  $\frac{6}{7} \div \frac{4}{5} - \frac{3}{7}$  of  $1\frac{1}{5} + \frac{5}{9}$  of  $\frac{18}{25} \div (\frac{2}{3} + \frac{6}{7}) \times \frac{4}{7}$ .

In 11 and similar examples the order of reduction is as follows:

1. Find the value of the fractions in brackets.
2. Simplify the compound fractions.
3. Invert each fraction before which the sign of division occurs.
4. Perform the multiplications indicated.
5. Perform the additions and subtractions in the order in which they occur.

12.  $\frac{6}{7} \div \frac{2}{3}$  of  $\frac{4}{5} = \frac{6}{7} \div \frac{8}{15}$ .
13.  $\frac{6}{7} \div \frac{2}{3} \times \frac{4}{5} = \frac{6}{7} \times \frac{3}{2} \times \frac{4}{5}$ .

## COMPLEX FRACTIONS

Since every fraction expresses the quotient of the numerator divided by the denominator, the division of one whole number by another may always be expressed in the form of a fraction. Thus, if it is required to divide 376 by 195, the operation may be indicated in the form of a fraction, thus:

$$\frac{376}{195}.$$

The same method is employed to indicate the division of one fraction by another;  $7\frac{1}{2} \div \frac{4}{5}$  being expressed in the form of a fraction, thus:

$$\frac{7\frac{1}{2}}{\frac{4}{5}}.$$

Also  $(\frac{3}{4} + \frac{2}{5}) \div \frac{2}{3}$  of  $6\frac{3}{5}$  may be written

$$\frac{\frac{3}{4} + \frac{2}{5}}{\frac{2}{3} \text{ of } 6\frac{3}{5}}.$$

When the division of one fraction by another is indicated in this way, the resulting expression is called a **Complex Fraction**, and the two fractions so used are called respectively the numerator and denominator of the complex fraction. Thus in the above fraction  $\frac{3}{4} + \frac{2}{5}$  is called the numerator, and  $\frac{2}{3}$  of  $6\frac{3}{5}$  the denominator.

Complex fractions may be reduced to simple ones by performing the division indicated, thus:

$$\begin{aligned} \frac{\frac{3}{4} + \frac{2}{5}}{\frac{2}{3} \text{ of } 6\frac{3}{5}} &= (\frac{3}{4} + \frac{2}{5}) \div \frac{2}{3} \text{ of } 6\frac{3}{5} \\ &= \frac{23}{20} \div \frac{22}{5} \\ &= \frac{23}{20} \times \frac{5}{22} \\ &= \frac{23}{88}. \end{aligned}$$

## EXERCISE

Simplify:

1.  $\frac{\frac{8}{9}}{\frac{5}{7}}$

2.  $\frac{3\frac{4}{5}}{\frac{2}{3}}$

3.  $\frac{\frac{7}{5}}{\frac{5}{6}}$

4.  $\frac{\frac{7}{5}}{\frac{6}{6}}$

5.  $\frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{2} + \frac{1}{3}}$

6.  $\frac{\frac{2}{3} \text{ of } \frac{4}{5} \times 2}{\frac{4}{5} \text{ of } 3\frac{1}{2}}$

7.  $\frac{\frac{3}{7} \times \frac{14}{7}}{\frac{5}{9} \div \frac{20}{27}}$

8.  $\frac{\frac{6}{7} \times \frac{5}{6} \div 7}{\frac{1}{9} + \frac{2}{3} \times \frac{6}{7}}$

9.  $\frac{\frac{2}{3}}{\frac{5}{6}} + \frac{\frac{3}{5}}{\frac{7}{8}}$

10.  $\frac{7\frac{1}{2}}{3\frac{1}{6}} + \frac{\frac{2}{3} \text{ of } 6}{\frac{1}{5} \text{ of } 4}$

11.  $\frac{\frac{6}{7} + \frac{2}{9}}{\frac{2}{3} - \frac{1}{6}} + \frac{\frac{2}{3} + \frac{1}{6}}{\frac{6}{7} - \frac{2}{9}}$

## GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE OF FRACTIONS

The G. C. M. of 9, 12 and 15 is 3, and the G. C. M. of 9 quarts, 12 quarts and 15 quarts is 3 quarts.

In the same way the G. C. M. of 9 seventeenths, 12 seventeenths and 15 seventeenths is 3 seventeenths. That is, the G. C. M. of

$$\frac{9}{17}, \frac{12}{17} \text{ and } \frac{15}{17} \text{ is } \frac{3}{17}.$$

So that if any number of fractions have the same denominator their G. C. M. is found by taking the G. C. M. of their numerators, and placing under it the common denominator.

If the fractions have not the same denominator they can be reduced to equivalent fractions which have a common denominator, and then their G. C. M. can be found in the manner already stated.

It is evident that the L. C. M. of any number of fractions could be found in a similar manner, by reducing

them to a common denominator, and then finding the L. C. M. of the new numerators, and dividing it by the common denominator.

A simpler method of finding the G. C. M. and L. C. M. of any number of fractions may be obtained in the following manner:

Take any two fractions,  $\frac{14}{5}$  and  $\frac{21}{10}$ , which are in their lowest terms.

Any fraction which will divide an integral number of times into  $\frac{14}{5}$  must have its numerator a factor of 14, and its denominator a multiple of 15.

Thus  $\frac{14}{5} \div \frac{2}{3} = \frac{14}{5} \times \frac{3}{2}$ , and is therefore not an integer because 15 does not divide into 3.

But  $\frac{14}{5} \div \frac{7}{60} = \frac{14}{5} \times \frac{60}{7}$ , which is an integer, since 7 is a factor of 14 and 60 is a multiple of 15.

It follows then, that any common measure of  $\frac{14}{5}$  and  $\frac{21}{10}$  must have its numerator a common factor of 14 and 21, and its denominator a common multiple of 15 and 40.

To be the **greatest** common measure, the numerator must be as large as possible, and the denominator as small as possible.

It therefore follows that the numerator of the G. C. M. must be the G. C. M. of the numerators of the given fractions, and the denominator of the G. C. M. must be the L. C. M. of the denominators:

$\therefore$  the G. C. M. of  $\frac{14}{5}$  and  $\frac{21}{10}$

$$= \frac{\text{G. C. M. of 14 and 21}}{\text{L. C. M. of 15 and 40}} = \frac{7}{120}.$$

To find the L. C. M. of  $\frac{2}{15}$ ,  $\frac{5}{6}$  and  $\frac{4}{9}$ .

Any fraction into which  $\frac{2}{15}$  will divide an integral number of times must have its numerator a multiple of 2, and its denominator a factor of 15.

Hence any common multiple of  $\frac{2}{15}$ ,  $\frac{5}{6}$  and  $\frac{4}{9}$  must have its numerator a common multiple of 2, 5 and 4, and its denominator a common factor of 15, 6 and 9.

To be the least common multiple, the numerator must be as small as possible, and the denominator as large as possible.

It therefore follows that the numerator of the L. C. M. is the L. C. M. of the numerators of the fractions, and the denominator of the L. C. M. is the G. C. M. of the denominators:

$\therefore$  the L. C. M. of  $\frac{2}{15}$ ,  $\frac{5}{6}$  and  $\frac{4}{9}$

$$= \frac{\text{L.C.M. of 2, 5 and 4}}{\text{G.C.M. of 15, 6 and 9}} = \frac{20}{3}.$$

NOTE.—In finding the G. C. M. or L. C. M., by the above methods, the fractions must first be reduced to their lowest terms.

## Decimals and Decimal Fractions

---

In the decimal system of notation the value of a digit in any place is always ten times the value of the same digit written in the next place to the right, or one tenth of the value of the same digit in the next place to the left.

Thus in the number 444,  
the 4 in the hundreds' place represents 4 hundreds, or 400 units,  
the 4 in the tens' place represents 4 tens, or 40 units,  
the 4 in the units' place represents 4 units.

If now another 4 is placed to the right of the units' place, it should represent

$$\frac{1}{10} \text{ of } 4 \text{ or } \frac{4}{10}$$

The number would then be written 444.4.

The dot which is placed to the right of the units' digit is called the **Decimal Point**.

The 4 to the right of the decimal point is read 4 tenths.

If another 4 were written to the right of the 4 tenths it would represent  $\frac{1}{10}$  of  $\frac{4}{10}$  or  $\frac{4}{100}$ , and is read 4 hundredths.

The next digit to the right would represent a number of thousandths, the next a number of ten-thousandths, and so on.

If a number does not contain any units, or any higher denomination, it may be written

$$0.376,$$

where the cipher takes the units' place, but it is more frequently written

$$.376,$$

and denotes 3 tenths, 7 hundredths, and 6 thousandths.

If there are no tenths in a number, a 0 takes its place, as

$$.076.$$

In the same way we may have

$$.003, \quad .00004, \text{ and so on;}$$

the last denotes 4 hundred-thousandths.

Such numbers are called **Decimals, or Decimal Fractions.**

The number

$$.67$$

denotes 6 tenths and 7 hundredths; that is,

$$\frac{6}{10} \text{ and } \frac{7}{100},$$

$$\text{but } \frac{6}{10} \text{ and } \frac{7}{100} = \frac{6}{10} + \frac{7}{100} = \frac{67}{100}.$$

Therefore .67 is read *sixty-seven hundredths*.

In the same way .347 is read *three hundred and forty-seven thousandths*. Thus, every decimal is read as a whole number of the denomination indicated by the last digit to the right.

The number

$$396.89$$

is read three hundred and ninety-six, and eighty-nine hundredths.

## EXERCISE

Write the following numbers in words:

- |               |                 |               |               |
|---------------|-----------------|---------------|---------------|
| 1. 7.6.       | 2. 39.3.        | 3. 4.89.      | 4. 762.       |
| 5. .762.      | 6. 762.762.     | 7. 1234.5678. | 8. 123.45678. |
| 9. 2400.0036. | 10. .2436.      | 11. .0006.    |               |
| 12. .000006.  | 13. .000000006. |               |               |

Express in digits:

14. Seventy-six and eighty-nine hundredths.
15. Fourteen and three thousandths.
16. One hundred, and three ten-thousandths.
17. One hundred and three ten-thousandths.
18. Thirty thousand and seventy, and one thousand and eighty-three millionths.

In practice 37.34056 is read "37, decimal, 3, 4, 0, 5, 6,"  
 .7854 is read "decimal, 7, 8, 5, 4."

Since .6 is six tenths =  $\frac{6}{10}$ ,

.69 is sixty-nine hundredths =  $\frac{69}{100}$ ,

.347 =  $\frac{347}{1000}$ , and .03 =  $\frac{3}{100}$ ,

therefore it appears that a decimal may be expressed in the form of a fraction by taking the decimal, after removing the point, for numerator; and for denominator, 1 followed by as many 0's as there are digits to the right of the decimal point.

And conversely any fraction whose denominator is 1 followed by 0's may be expressed as a decimal by omitting the denominator and placing the decimal point in the numerator in such a way that there will be as many

digits to the right of it as there were 0's in the denominator.

$$\text{Thus } \frac{347}{1000} = .347.$$

$$\frac{347}{10000} = .0347.$$

In this last example, since there are 4 0's in the denominator and only 3 digits in the numerator, a cipher is placed between the decimal point and the first digit of the numerator.

In the same way,

$$\frac{7}{1000} = .007.$$

## ADDITION AND SUBTRACTION OF DECIMALS

The addition and subtraction of decimals are performed in the same way as the addition and subtraction of integers.

In arranging the numbers to perform either of these operations care must be taken to place those digits which have the same denomination under each other. Thus units must be placed under units, tenths under tenths, and so on.

This is most easily done by placing the numbers so that the decimal points will be directly under each other.

To add .3487, 16.396, 3.04324 and 746.28, arrange the numbers thus:

$$\begin{array}{r} .3487 \\ 16.396 \\ 3.04324 \\ 746.28 \\ \hline 766.06794 \end{array}$$

Subtract 3.14159265 from 57.29577. Arrange the numbers thus:

$$\begin{array}{r} 57.29577000 \\ 3.14159265 \\ \hline 54.15417735 \end{array}$$

NOTE.—The three ciphers which have been added to the minuend do not change its value, and therefore do not affect the result.

### EXERCISE

Add together:

1. 36.4158, 3426, .3246, 59.637 and 3.6.
2. 57.29577, .01, .7854, 1.732 and 1.4142.
3. 3.14159265, 2500, .4771213 and .00004848.
4. Subtract 5.239476 from 100.462.
5. Subtract the sum of 3.497 and 2.1685 from 10.
6. Find the value of  $10.3 - 4.267 + .64 - .0976$ .

### MULTIPLICATION OF DECIMALS

If the decimal point in a number be moved one place to the right the number will be multiplied by 10.

Take the number 67.9428 and remove the decimal point one place to the right and the result is

679.428.

Comparing this with the original number

8 ten-thousandths	has become	8 thousandths,
2 thousandths	“ “	2 hundredths,
4 hundredths	“ “	4 tenths,
9 tenths	“ “	9 units,
7 units	“ “	7 tens,
6 tens	“ “	6 hundreds,

so that each digit represents ten times as much in the new number as in the old, and therefore the new number is ten times the old one.

If the decimal point be again moved one place to the right the result will be 100 times the original number; 3 places, 1000 times, and so on.

It will be evident, too, that if the decimal point be moved one place to the left the number will be divided by 10; if two places, by 100; if three places, by 1000, and so on.

In the case of a pure decimal the decimal point is moved to the left by placing ciphers between the point and the nearest figure, thus: .67 when divided by 10 becomes .067; and this when divided by 10 becomes .0067, and so on.

Multiply 37.643 by 3.86.

$$\begin{aligned}
 37.643 &= 37 \frac{643}{1000} = \frac{37643}{1000}; \\
 \text{and } 3.86 &= 3 \frac{86}{100} = \frac{386}{100}. \\
 \therefore 37.643 \times 3.86 &= \frac{37643}{1000} \times \frac{386}{100} \\
 &= \frac{37643 \times 386}{100000} \\
 &= \frac{14530198}{100000} \\
 &= 145.30198
 \end{aligned}$$

Now this result, if the decimal point be omitted, is the product of 37643 and 386, which are the multiplicand and multiplier with their decimal points removed, and the number of digits after the decimal point in the product is equal to the sum of those in the multiplicand and multiplier, there being 3 in the multiplicand, 2 in the multiplier, and 5 in the product.

It will sometimes be necessary to place ciphers to the left of the figures in the product in order to obtain the requisite number of decimal figures, thus,

$$.3 \times .2 = \frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = .06.$$

Here the product of 3 and 2 consists of one digit only, but the product requires two decimal digits, which are obtained by the introduction of the cipher.

### EXERCISE

Find the product of:

1. 1.732 and 1.4142.
2. 2.236 and 2.4495.
3. 147.4771 and .30103.
4. 57.29577 and 1.01.
5. 20403.1416 and .000378.
6. .00004848 and 5463.
7. 101.0101, 37.203 and 3.2.
8. .04, .06, .003 and .1.
9. .1, .01, .001, .0001 and 1000.
10. .1, .1, .1, .01, .01, .01 and 1,000,000,000.

### DIVISION OF DECIMALS

If it is required to divide 57.20575 by 6.79, the division is performed in the same way as if the numbers were integers, the only difference being in the interpretation of the result.

When 5720575 is divided by 679, the result is 8425.

When 57.20575 is divided by 6.79 the first digit in the quotient, 8, is evidently a number of units, since 6 when divided into 57 gives but one digit in the integral part. If 8 is a units' digit the quotient must be 8.425.

Divide  $5.720575$  by  $.0679$ .

Multiply both divisor and dividend by 100, with the object of getting one digit only in the integral part of the divisor.

This operation will evidently leave the quotient unchanged.

We have now to divide  $572.0575$  by  $6.79$ .

There will now be 2 digits in the integral part of the quotient, and therefore the quotient is  $84.25$ .

Divide  $.05720575$  by  $67.9$ .

Move the decimal point one place to the left in each, which is equivalent to dividing each by 10.

Then we have to divide  $.005720575$  by  $6.79$ .

It is now seen that the first significant digit in the quotient is in the fourth decimal place, and therefore the result is  $.0008425$ .

In dividing any two decimals then, first move the decimal points the same number of places in the dividend and the divisor, so that the divisor will have one digit in the integral part. The place of the first significant digit in the quotient may then be determined by inspection, and after the division is performed, the decimal point may be properly placed in the quotient.

### EXERCISE

Divide:

1.  $1.728$  by  $1.44$ , by  $14.4$  and by  $144$ .
2.  $4.096$  by  $10.24$ , by  $102.4$ , by  $25.6$ , by  $.32$ , by  $.064$ .

3. 53.1441 by 6.561, by 65.61, by 59.049, by 19.683, by .243, and by 177147.

4. .390625 by 25, by .005, by .000625, by 15625, by .625.

5. 177.1561 by 146410, 1100, 12100, 13310.

6. 34.2 by 25, 625, 12.5, 250, 500.

7. 16976.16 by .006, .216, .1296, .07776.

8. 1 by 2, 5, 8, 25, 64, 125, 256, 625.

Divide to 5 places of decimals:

9. 1.09861 by 2.302585.

10. 1 by 2.31258.

11. .30103 by .43429.

12. 113 by 355.

## TO CONVERT A DECIMAL INTO A VULGAR FRACTION

Since .9375 may be expressed as

$$\frac{9375}{10000},$$

this fraction may be reduced to its lowest terms in the manner already indicated. Thus if we divide successively

by 5 we obtain  $\frac{15}{16}$ .

$$\text{Similarly } .5104 = \frac{5104}{10000} = \frac{319}{625}.$$

## EXERCISE

Reduce to equivalent vulgar fractions:

1. .96875, .8125, .0064, .00032.

It will be observed in reducing these decimals that no divisors but 2 and 5 or some powers of them are used, the reason being that 10 and any power of 10, as 100, 1000, etc., have no factors but 2 and 5, or powers of 2 and 5.

## TO CONVERT A VULGAR FRACTION TO A DECIMAL

When 11 is divided by 16 the result is  $\cdot 6875$ .

But  $11 \div 16$  is equivalent to the fraction  $\frac{11}{16}$ .

$$\therefore \frac{11}{16} = \cdot 6875.$$

## EXERCISE

Reduce to equivalent decimals:

$$1. \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$$

$$2. \frac{3}{16}, \frac{5}{16}, \frac{7}{16}, \frac{9}{16}, \frac{11}{16}, \frac{13}{16}, \frac{15}{16}$$

$$3. \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$$

$$4. \frac{24}{25}, \frac{124}{125}, \frac{624}{625}$$

$$5. \frac{31}{32}, \frac{63}{64}, \frac{127}{128}, \frac{1}{256}$$

Express as decimals to 6 places:

$$6. \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$$

$$7. \frac{34}{37}, \frac{25}{37}, \frac{59}{111}, \frac{146}{333}, \frac{386}{999}$$

$$8. \frac{1}{3}, \frac{2}{3}, \frac{1}{6}, \frac{5}{6}, \frac{7}{9}, \frac{8}{11}, \frac{9}{13}$$

## CIRCULATING DECIMALS

If we reduce the fractions  $\frac{7}{8}$  and  $\frac{5}{6}$  to decimals, we get for the former  $\cdot 875$  and for the latter  $\cdot 8333$ , with remainder 2, which will give another 3 in the quotient and another remainder 2, and so on forever. Such a decimal as  $\cdot 8333$  etc., is called a **Repeating or Circulating Decimal**. The repeating digits may occur either singly or in groups,

and may either commence immediately after the decimal point or at some distance from it.

Thus:  $\frac{1}{3} = .333$  etc., which is written  $\dot{.}3$ .

$$\frac{1}{6} = .16666 \text{ etc.,} \quad \text{“} \quad \text{“} \quad \dot{.}1\dot{6}.$$

$$\frac{11}{12} = .916666 \text{ etc.,} \quad \text{“} \quad \text{“} \quad .91\dot{6}.$$

$$\frac{8}{11} = .72727272 \text{ etc.,} \quad \text{“} \quad \text{“} \quad \dot{.}7\dot{2}.$$

$$\frac{19}{37} = .513513513 \text{ etc.,} \quad \text{“} \quad \text{“} \quad \dot{.}51\dot{3}.$$

$$\frac{17}{44} = .38636363 \text{ etc.,} \quad \text{“} \quad \text{“} \quad .38\dot{6}\dot{3}.$$

Every terminating decimal can be expressed as a vulgar fraction whose denominator is a power of 10.

$$\text{Thus, } .32 = \frac{32}{100}, \quad .019 = \frac{19}{1000}, \text{ etc.}$$

Conversely, every vulgar fraction having a denominator which is a power of 10, or which can be changed into such a fraction, can be expressed as a terminating decimal.

$$\text{Thus: } \frac{3}{10} = .3$$

$$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = .8$$

$$\frac{3}{4} = \frac{3}{2 \times 2} = \frac{3 \times 5 \times 5}{2 \times 2 \times 5 \times 5} = \frac{75}{100} = .75.$$

But it is evident that a fraction, in its lowest terms, cannot be changed to an equivalent fraction whose denominator is a power of 10, if its denominator contains any prime factors other than 2 or 5.

Thus  $\frac{3}{7}$ ,  $\frac{5}{14}$ ,  $\frac{7}{30}$  cannot be reduced to terminating decimals because 7, 14 and 30 respectively contain other prime factors besides 2 and 5.

It follows then, that if the denominator of a fraction (in its lowest terms) contains factors other than 2 and 5, the corresponding decimal can never terminate.

We have next to show that these non-terminating decimals must repeat.

$$\begin{array}{r} 7 \overline{) 3.000000000000} \\ \underline{.428571428571} \end{array}$$

Reducing the fraction  $\frac{3}{7}$  to a decimal, we find that the remainders in order are

2, 6, 4, 5, 1, 3,

and the dividends must therefore be (after the first)

20, 60, 40, 50, 10, 30.

Now since the last of these is the same as the dividend with which we started,

therefore the quotient obtained from it will be the same as the first quotient;

therefore the remainder obtained from it will be the same as the first remainder, and so on;

therefore the second set of 6 digits in the quotient must be the same as the first set of 6 digits; and for the same reason the third set will be the same as the second.

Thus the digits in the quotient must recur in the same order.

And since in dividing by seven there can be only 6 different remainders, it follows that there can be only 6 digits in the repeating part of the decimal. Similarly, if

the denominator of the fraction is 17, there cannot be more than 16 digits in the repeating part; if the denominator is 19, not more than 18 digits, and so on. So that the number of digits in the repeating part of a decimal cannot be greater than the divisor diminished by 1.

The number of digits in the repeating part will not always be so great as this, for if the denominator be 3, 6, or 9, only one digit will repeat; if 11, two digits; 13, six digits; 31, fifteen digits; 37, three digits; 41, five digits.

To determine where the decimal will begin to repeat:

$$\begin{array}{lll} \text{We find that} & \frac{1}{3} = .\dot{3}. \\ \text{"} & \text{"} & \frac{1}{6} = .1\dot{6}. \\ \text{"} & \text{"} & \frac{7}{12} = .58\dot{3}. \\ \text{"} & \text{"} & \frac{19}{24} = .791\dot{6}. \end{array}$$

In the first of these the factor 2 does not occur in the denominator, and the decimal begins to repeat at the decimal point.

In the second case the factor 2 occurs once in the denominator and one digit occurs between the decimal point and the repeating digit.

In the third case two 2's occur as factors in the denominator and two digits occur between the decimal point and the repeating digit.

In the fourth, three, and so on, every 2 which occurs as a factor in the denominator having the effect of moving the first repeating digit one place further to the right, and the same will be found to be true if 5 be substituted for 2 in what has just been stated.

If 2's and 5's both occur in the denominator, the same thing will be true of the one that occurs oftener, and the

other will not affect the position of the first repeating digit. Thus if 2 occurs four times, and 5 three times, there will be four intervening digits; if 5 occurs three times and 2 once, there will be 3 intervening digits.

The reduction of a vulgar to a decimal fraction may generally be much shortened, as in the following examples:

1. Reduce  $\frac{7}{19}$  to a decimal fraction.

$$\begin{array}{r}
 19 \overline{)70}(\cdot 36842105263157894\dot{7}36 \\
 \underline{57} \\
 130 \\
 \underline{114} \\
 160 \\
 \underline{152} \\
 80
 \end{array}$$

It is evident that the result obtained by dividing 19 into 80 will be half that obtained by dividing 19 into 160. We may therefore begin at the figure obtained from 160 as dividend and divide by 2, and continue this operation. Thus divide 2 successively into 8, 4, 2, 1, 10, 5, 12, 6, etc., and obtain the quotients 4, 2, 1, 0, 5, 2, 6, etc.

2. Reduce  $\frac{12}{13}$  to a decimal.

$$\begin{array}{r}
 13 \overline{)120}(\cdot 92307\dot{6}9 \\
 \underline{117} \\
 3
 \end{array}$$

Here, since 3 is  $\frac{1}{4}$  of 12, we may at once divide 4 into 9, 12, 3, 30, 27, 36, etc.

Thus, in the operation of reducing a vulgar fraction to a decimal, when a remainder is reached which is an exact divisor of any previous remainder, we may commence dividing at the digit obtained as quotient to that previous remainder.

## EXERCISE

Reduce to decimal fractions:

$$1. \frac{18}{19}, \frac{12}{19}, \frac{10}{17}, \frac{6}{17}, \frac{8}{13}, \frac{9}{23}, \frac{8}{21}.$$

$$2. \frac{14}{23}, \frac{13}{29}, \frac{30}{31}, \frac{36}{37}, \frac{1}{41}, \frac{19}{53}.$$

$$3. \frac{80}{91}, \frac{17}{63}, \frac{76}{77}, \frac{100}{259}, \frac{98}{189}, \frac{1}{2849}, \frac{1000}{1001}.$$

If we reduce  $\frac{1}{7}$ ,  $\frac{2}{7}$ ,  $\frac{3}{7}$ ,  $\frac{4}{7}$ ,  $\frac{5}{7}$ ,  $\frac{6}{7}$  to decimals, we have

$$\frac{1}{7} = .\dot{1}4285\dot{7}, \quad \frac{2}{7} = .\dot{2}8571\dot{4}, \quad \frac{3}{7} = .\dot{4}2857\dot{1},$$

$$\frac{4}{7} = .\dot{5}7142\dot{8}, \quad \frac{5}{7} = .\dot{7}1428\dot{5}, \quad \frac{6}{7} = .\dot{8}5714\dot{2}.$$

In these cases the same digits occur in all the decimals, and in the same order. Thus, 1 is always followed by 4; 4, by 2; 2, by 8, and so on.

The following examples furnish illustrations of the application of this result:

1. Knowing that  $\frac{1}{7} = .\dot{1}4285\dot{7}$ , find the decimal equivalent to  $\frac{5}{7}$ .

By division we get 7 as the first digit in the quotient; then, knowing that 7 is followed by 1, 1 by 4, etc., we have at once 714285, and therefore

$$\frac{5}{7} = .\dot{7}1428\dot{5}.$$

2. Reduce  $\frac{27}{8}$  to a circulating decimal.

$$28)270(.964$$

$$\underline{252}$$

$$180$$

$$\underline{168}$$

$$120$$

$$\begin{aligned} \therefore \frac{27}{8} &= .96\frac{1}{2}\frac{2}{8} \\ &= .96\frac{3}{4} \end{aligned}$$

In this case, since there are two 2's as factors of the denominator, there will be two digits between the decimal point and the first of the repeating digits. Therefore 4 is the first repeating digit, and the others are known at once to be 2, 8, 5, 7, 1, and therefore the required result is  $\cdot 96428571$ .

3. Reduce  $\frac{1}{175}$  to a circulating decimal.

Here the factors of the denominator are 5, 5, 7; therefore there will be two digits between the decimal point and the repetend, and the repetend will contain six digits. Hence the same method as in Ex. 2, gives the result  $\cdot 00571428$ .

From the value of  $\frac{7}{19}$  already found, we can find the value of any other proper fraction having 19 for denominator. Take for example  $\frac{3}{19}$ :

On dividing, the first digit in the quotient is 1, but as 1 occurs twice in the repetend, we must obtain by division another digit in the quotient; this is 5. Then we are at once enabled to write down the remaining digits of the repetend, namely, 789, etc., and therefore the required result is  $\cdot 157894736842105263$ .

## TO REDUCE CIRCULATING DECIMALS TO VULGAR FRACTIONS

$$\cdot \dot{2} = \cdot 2222 \dots$$

$$\therefore 10 \text{ times } \cdot \dot{2} = 2 \cdot 2222 \dots$$

$$\therefore 9 \text{ times } \cdot \dot{2} = 2$$

$$\therefore \cdot \dot{2} = \frac{2}{9}.$$

$$\text{Similarly } \cdot \dot{4} = \frac{4}{9}, \quad \cdot \dot{7} = \frac{7}{9}, \text{ and so on.}$$

$$\dot{.2\dot{3}} = .232323\dots$$

$$\therefore 100 \text{ times } \dot{.2\dot{3}} = 23.232323\dots$$

$$\therefore 99 \text{ times } \dot{.2\dot{3}} = 23$$

$$\therefore \dot{.2\dot{3}} = \frac{23}{99}.$$

Similarly  $\dot{.4\dot{7}} = \frac{47}{99}$ ,  $\dot{.24\dot{6}} = \frac{246}{999}$ , and so on.

$$\dot{.3872\dot{5}} = .38725725\dots$$

$$\therefore 100 \text{ times } \dot{.3872\dot{5}} = 38.725725\dots$$

$$\therefore 100,000 \text{ times } \dot{.3872\dot{5}} = 38725.725725\dots$$

$$\therefore (100,000 - 100) \text{ times } \dot{.3872\dot{5}} = 38725 - 38$$

$$\begin{aligned} \therefore \dot{.3872\dot{5}} &= \frac{38725 - 38}{100000 - 100} \\ &= \frac{38687}{99900} \end{aligned}$$

The numerator of this fraction is seen to be the difference between the given decimal, to the end of the first period, and the part of the decimal which does not repeat.

The denominator consists of as many nines as there are digits in the repeating period followed by as many ciphers as there are digits between the decimal point and the first of the repeating digits.

$$\text{Thus } \dot{.0326\dot{7}} = \frac{3267 - 32}{99000}.$$

$$12.\dot{347\dot{5}} = \frac{123475 - 123}{9990}.$$

### EXERCISE

Reduce to vulgar fractions in their lowest terms:

1.  $\dot{.6}$ ,  $\dot{.72}$ ,  $\dot{.09}$ ,  $\dot{.387}$ ,  $\dot{.6021}$ ,  $\dot{.714285}$ .

2.  $\dot{.194\dot{5}}$ ,  $\dot{.09}$ ,  $\dot{.0208\dot{3}}$ ,  $\dot{.114285\dot{7}}$  or  $\dot{.1\dot{7}}$ .

3.  $2.\dot{3}$ ,  $3.\dot{16}$ ,  $14.\dot{03\dot{7}}$ ,  $7.\dot{1457142\dot{8}}$ .

## ADDITION AND SUBTRACTION OF CIRCULATING DECIMALS

1. When absolute accuracy is not required.

A result sufficiently correct for most purposes may be obtained in the manner shown in the following example:

Add together the following numbers so as to obtain their sum correct to 4 decimal places:

$$.3\bar{7}, .2\bar{3}\bar{5}, .3514285\bar{7}, .\bar{7}, .0\bar{3}\bar{6}.$$

Extend each decimal to 6 places—2 more than the required number, thus:

$$\begin{array}{r} .373737 \\ .235353 \\ .351428 \\ .777777 \\ .036363 \\ \hline 1.774658 \end{array}$$

The difference between this result and the true result is less than one ten-thousandth. That is, it differs from the true result by less than 1 in the fourth decimal place.

### EXERCISE

Add:

1.  $.7, .7\bar{3}, .84\bar{6}, .934\bar{2}, .6320\bar{5}$  to 4 places.
2.  $.6\bar{3}, .06\bar{4}, .006\bar{7}, .937\bar{6}, .040\bar{6}$  to 4 places.
3.  $.734\bar{2}, .9476\bar{5}, .37641\bar{0}, .100100\bar{0}, 38.7\bar{4}$  to 5 places.
4.  $106.3\bar{0}, 937.4, 89.001, 387.159\bar{1}, 101.10\bar{1}, 1000.1000\bar{1}$  to 6 places.

2. When absolute accuracy is required.

We may either reduce the decimals to their equivalent

vulgar fractions and then add—which is apt to be a tedious operation—or we may proceed as follows:

Required to add  $\dot{.36}$ ,  $\dot{.539}$ ,  $\dot{.032437}$ .

*First* make the repeaters all *begin* at the same distance from the decimal point.

Now, since  $\dot{.36} = .36363636\dots$ , it may be written in any of the following ways:

$\dot{.363}$   
 $\dot{.3636}$   
 $\dot{.3636}$   
 $\dot{.3636363}$   
 $\dot{.363636363}$ , etc.

For all of these give the same result, namely,

$\dot{.3636363636\dots}$

Thus the place where the repeater begins may be moved to the right. We must therefore select that decimal which has its first repeating digit farthest to the right, as our guide in arranging the others.

This is  $\dot{.032437}$ ,  
 therefore the others become  $\dot{.36363}$ ,  
 and  $\dot{.53939}$ .

*Secondly*, we must make the repeaters all *end* at the same number of places from the decimal point.

The first of these must take some such form as,

$\dot{.032437}$   
 $\dot{.032437437}$   
 $\dot{.032437437437}$

That is, the number of digits in the repeater must be a multiple of 3.

Similarly in the other two the number of digits in the repeater must be a multiple of 2, and since 6 is a multiple of both 3 and 2, we may have 6 digits in the repeater in each case. Thus these numbers become

$$\begin{array}{r|l}
 .032 & \dot{4}3743\dot{7} \\
 .363 & \dot{6}3636\dot{3} \\
 .539 & \dot{3}9393\dot{9} \\
 \hline
 .935 & \dot{4}6774\dot{0}
 \end{array}$$

We then proceed to add them together, remembering to increase the right-hand column by the number to be carried as the result of adding the digits which would be to the right of this if the decimals were extended.

### EXERCISE

Add:

1.  $\dot{.3}$ ,  $\dot{.265}$ ,  $\dot{.037}$ .
2.  $\dot{.36}$ ,  $\dot{.364}$ ,  $\dot{.9873}$ .
3.  $\dot{.000037}$ ,  $\dot{.4038}$ ,  $\dot{.9}$ ,  $\dot{.27}$ .
4.  $\dot{.398457}$ ,  $\dot{.00375}$ ,  $\dot{.00018}$ .

The method to be pursued in subtraction needs no further explanation.

## MULTIPLICATION AND DIVISION OF CIRCULATING DECIMALS

1. When absolute accuracy is required.

Reduce the decimals to their equivalent vulgar fractions:

Perform the required operation:

Reduce the resulting fraction to a decimal.

2. When absolute accuracy is not required.

The requisite degree of accuracy may be secured by the methods of contracted multiplication and division of decimals exemplified below.

### CONTRACTED MULTIPLICATION OF DECIMALS

Multiply 9.8413 by 1.234, obtaining the product correct to 3 decimal places.

(a)		(b)		(c)	
9.8413		9.8413		9.8413	
1.234		1.234		4321	
39	3652	9841	3	9841	3
295	239	1968	26	1968	3
1968	26	295	239	295	2
9841	3	39	3652	39	4
12.144	1642	12.144	1642	12.144	

In the foregoing methods of multiplication (a) shows the complete product obtained in the usual way, (b) also shows the complete product obtained by a method which differs from (a) only in beginning the operation with the first digit on the left of the multiplier. In the complete multiplication, if the result is required correct only to 3 decimal places, it is clear that the only use of the digits on the right of the vertical line, in (a) or (b), is to obtain the digit to be carried to the first column on the left of that line.

To obtain the proper digit to be carried, the first column on the right of the line is generally all that is necessary, so that all the other digits might be omitted.

It is only necessary to multiply the 1 of the multiplier into 98413, the 2 into 9841, the 3 into 984, and the 4 into 98.

The simplest way to do this, is to arrange the work

as shown in (c), then each digit of the multiplier is multiplied into the digit above it and those to the left.

Each digit of the multiplier must be multiplied into the first digit to the right in the multiplicand for the purpose of obtaining the digit to be carried.

Thus  $98 \times 4 = 392$ , but  $4 \times 4 = 16$ , from which 2 is carried, making the last product 394. Here 2 is carried because 16 is nearer to 20 than to 10. Similarly to the product of 984 by 3, 0 is carried, as the product  $1 \times 3$  is nearer to 0 than to 10. To the product  $9841 \times 2$ , 1 is to be carried.

In obtaining the digit to be carried the following rule should be followed: If the product is between 0 and 4 inclusive, carry 0, if between 5 and 14 inclusive, carry 1; if between 15 and 24 inclusive, carry 2, and so on.

If we wish to multiply to 4 decimal places, that is to obtain the product correct to 3 decimal places, the multiplier then is written in reversed order, the units' digit of the multiplier being placed under the digit in the fourth decimal place of the multiplicand.

Multiply .344657 by .356432 correct to 4 decimal places.

.3446	57
234653	0
0000	0
1034	0
172	3
20	6
1	3
	1
<hr/>	
.1228	

Here the units' digit of the multiplier is 0 and is placed under the digit in the fifth decimal place in the multiplicand. The multiplication by the 0 may, of course, be omitted in the work.

Multiply  $\cdot 053726$  by  $13\cdot 56432$  correct to 3 decimal places.

$$\begin{array}{r|l}
 \cdot 0537 & 26 \\
 234653 & 1 \\
 \hline
 537 & 3 \\
 161 & 2 \\
 26 & 9 \\
 3 & 2 \\
 & 2 \\
 \hline
 \cdot 728 & 
 \end{array}$$

Here the units' digit of the multiplier is placed under the fourth decimal place in the multiplicand, and the tens' digit of the multiplier under the fifth decimal place. In the multiplication by 5 we carry a 4 from the product 35, since the complete product would be more than 35.

When the multiplier does not contain **just one** digit in the integral part, as in the last two examples, it might be better to change both multiplier and multiplicand so that there would be **only one** digit in the integral part of the multiplier.

Thus  $\cdot 344657 \times \cdot 356432$  would become  
 $\cdot 0344657 \times 3\cdot 56432$ ,  
 and  $\cdot 053726 \times 13\cdot 56432$  would become  
 $\cdot 53726 \times 1\cdot 356432$ .

The operations here performed would evidently **leave the** product in each case unchanged.

### EXERCISE

1. Find the product of  $47\cdot 3846$  by  $3\cdot 14159$  correct to three decimal places.
2. Find  $(1\cdot 06)^5$  correct to three places.
3. Find the product of  $1\cdot 07$ ,  $1\cdot 035$ ,  $1\cdot 05$ ,  $1\cdot 025$  correct to four places.
4. Find  $\cdot 3725 \times \cdot 3725 \times 3\cdot 1416$  correct to three places.
5. Find  $4 \times 3\cdot 14159 \times (2\cdot 37)^2$  to four places.

## CONTRACTED DIVISION OF DECIMALS

Divide 234398766 by 23456789 to six digits in the quotient.

Omit the 9 in the divisor and divide to six digits. How far does this quotient coincide with the first?

Omit the 8 also and divide to six digits. How far is the quotient correct in this case?

If you divide by 23456 instead of the complete divisor, how far will the quotient be correct?

What relation do you notice between the number of digits in the divisor and the number of digits to which the quotient is correct in each case?

NOTE.—If in any case we divide by, say, the first seven digits of the divisor, the quotient will generally be correct to at least six digits. Thus in every case we have only to take as much of the divisor as will contain one digit more than the number required in the quotient.

Divide 2.718281828 by 3.14159265 correct to five digits in the quotient. Take the first six digits for divisor, thus:

$$\begin{array}{r} 314159\cancel{265})2718281828(8 \\ \underline{2513274} \\ 205007 \end{array}$$

As we have now to obtain four digits in the quotient we divide by five digits of the divisor, thus:

$$\begin{array}{r} 314159\cancel{265})205007(6 \\ \underline{188495} \\ 16512 \end{array}$$

Similarly we next divide by four digits, thus:

$$\begin{array}{r} 314159\cancel{265})16512(5 \\ \underline{15708} \\ 804 \end{array}$$

Next dividing similarly by three and two digits of the divisor we get the other two digits of the quotient, which is therefore 86525, the position of the decimal point being determined in the same way as in ordinary division of decimals. The student must remember to take account of the digit to be carried in each of the partial multiplications as has been already explained. The complete operation may be arranged thus:

$$\begin{array}{r}
 3.14159265)2.718281828(.86525 \\
 \underline{52568} \qquad \underline{2513274} \\
 \qquad \qquad \underline{205007} \\
 \qquad \qquad \underline{188495} \\
 \qquad \qquad \qquad \underline{16512} \\
 \qquad \qquad \qquad \underline{15708} \\
 \qquad \qquad \qquad \qquad \underline{804} \\
 \qquad \qquad \qquad \qquad \underline{628} \\
 \qquad \qquad \qquad \qquad \underline{176} \\
 \qquad \qquad \qquad \qquad \underline{157}
 \end{array}$$

A convenient arrangement is to place each digit in the quotient under the right hand digit of the divisor used in obtaining it; thus 8 was obtained by dividing by 314159, and is accordingly placed under 9; 6 is placed under 5 of its divisor 31415; and so on. This enables the multiplications by these digits to be readily effected, and also serves to indicate the next divisor to be used. The quotient, when obtained, may be arranged in proper order in its usual place, and the position of the decimal point determined.

In example (1) of the exercise following, five digits must be obtained in the quotient, as there will evidently be two digits in the integral part, and therefore we must begin the division with six digits in the divisor.

In example (2) we must begin with seven digits in the divisor, and a cipher should be placed to the right of it.

## EXERCISE

Find each of the following to three decimal places:

1.  $\frac{30}{1.00017}$

2.  $\frac{7000}{8.16834}$

3.  $\frac{83.365}{58.995}$

4.  $\frac{29.995}{1.001294}$

5.  $\frac{13631.361}{1002.766}$

6.  $\frac{110356.84}{9.815 \times 7990}$

7.  $\frac{1.04315}{1.0012} \times 252.286$

## EXERCISES IN FRACTIONS

1. Divide 3.003 by 148.28; .003003 by .014828; and 300.3 by 1.4828.

2. Find by the contracted method the product of 26.42783 and 5.23238 true to within .001.

3. Divide  $91.\dot{8}6\dot{3}$  by  $87.\dot{5}6$ .

4. Divide  $.14285\dot{7}$  by 5 without reducing to a vulgar fraction. Divide it also by 3.

5. Find the greatest and least values of a decimal the first four digits of which are known to be .8397.

6. By short division, find the quotient as a whole number and recurring decimal when 1769 is divided by 105.

7. Reduce nine and nine-tenths inches to the decimal of a mile

8. Find a decimal that shall be within  $\frac{1}{100000}$  of  $\frac{1}{4}\frac{1}{2}$ .

9. Simplify  $\frac{1}{2}$  of  $\frac{1}{3} \times \frac{3}{4}$  of  $\frac{4}{5} \div (\frac{1}{6} + \frac{1}{3}$  of 20).

10. What whole number of fiftieths most nearly expresses the value of  $\frac{876}{1169}$ ?

11. Find the least fraction which, added to the sum of  $\frac{7}{9}$ ,  $\frac{11}{13}$  and  $\frac{87}{90}$ , shall make the result an integer.

12. Which of the three numbers  $\frac{106}{39}$ ,  $\frac{2721}{1001}$  and 2.718282, most nearly expresses the value of 2.718281828.

13. Ascertain whether .52 or .519 more nearly represents the product of .834 and .623.

14. What number multiplied by 57.29577 will give a product differing from 180 by less than .0001?

15. Find the sum of:

$$(\frac{5}{7} + \frac{5}{2}), (\frac{5}{2} - \frac{5}{7}), (\frac{5}{7} \times \frac{5}{2}), (\frac{5}{7} \div \frac{5}{2}), (\frac{5}{2} \div \frac{5}{7}).$$

16. What fraction having 17 for numerator equals  $\frac{6\frac{1}{3}}{7}$ ?

17. Find, correct to four decimal places, a number which as a multiplier may be substituted for 3.14159 as divisor.

18. A vulgar fraction has 209 for its numerator and its nearest approximate value in thousandths is .511; what is its denominator?

19. State which of the following fractions will reduce to terminating and which to circulating decimals, and the limits to the number of digits in the repeating periods of the latter:

$$\frac{67}{128}, \frac{28}{37}, \frac{155}{1147}, \frac{51}{102}, \frac{47}{98}, \frac{321}{375}$$

20. Water expands one-tenth in freezing. If a cubic foot of water weighs 1000 oz., find the weight of a cubic foot of ice.

21. If 4 men in 5 days of 9 hours each can mow 15 acres of grass, how many men will mow 11 acres in 2 days of 11 hours each?

22. A person sold A  $\frac{3}{4}$  of his land, B  $\frac{4}{5}$  of the remainder, C  $\frac{5}{6}$  of what then remained and received \$50 for what he had left, at \$60 per acre. Find the number of acres he had at first.

23. Divide \$12.46 among A, B, C and D, so that A's share may be  $\frac{3}{10}$  of D's, C's share  $\frac{3}{10}$  of A's, and B's share the sum of A's and C's.

24. Find the average, correct to 4 places of decimals, of

$$12\frac{1}{2}, 21, 7\frac{3}{4}, .034, 3.125, 0, 24.58 \text{ and } 12\frac{9}{10}.$$

25. If 12 men, working 8 hours a day, do  $\frac{4}{5}$  of a piece of work in 20 days, how many days will 15 men, working 10 hours a day, take to do  $\frac{7}{8}$  of it?

26. A merchant fails owing \$10500, and pays his creditors 35 cents on the dollar. How much will be lost by a creditor to whom one-fifth of the debt was owing?

27. From a cask of wine, worth 90 cents a gallon, a sixth part is drawn and replaced by wine worth only 50 cents a gallon. What is now the value per gallon of the wine in the cask?

28. A sold a town lot to B and gained one-eighth, B sold it to C for \$306 and lost three twentieths. How much did the lot cost A?

29. A can beat B by 5 yards in a 100 yard race, and B can beat C by 10 yards in a 200 yard race; by how much can A beat C in a 200 yard race?

30. A grocer bought 600 lb. of sugar for \$52.10; he used 65 lb. himself and sold the rest so as to make  $1\frac{1}{2}$  cents per lb. profit on the whole quantity. How much per lb. did he sell it for?

31. A railway train has a journey of 65 miles to perform and usually makes the distance in 3 hours. If the train starts 15 min. late, how much must it increase its speed to arrive on time?

32. A house and lot are together worth \$2100; one-fourth of the value of the house is equal to one third of the value of the lot; find the value of each.

33. If three fourths of the price of a loaf of bread depends upon the price of flour, by how much should the price of a 10 cent loaf be increased when flour rises two fifteenths in value?

34. The sum of \$3276 is to be divided among four persons in proportion to their ages, which are 17, 20, 24 and 30 years. How would you divide it?

35. The sum which will pay A's wages for  $61\frac{1}{4}$  days, will pay B's wages for  $81\frac{3}{4}$  days; for how many days will it pay the wages of A and B together?

36. A grain dealer buys 5225 bushels of wheat at \$1.05 per bushel, and pays \$125 for insurance, storage, etc.; he sells  $\frac{2}{5}$  of it at 97 cents a bushel; at what price per bushel must he sell the remainder to gain \$522.50 on the whole?

37. A can do a piece of work in 10 days, B in 15 days, and C in 20 days. What fraction of the work can each do in 1 day? In how many days can they do the work if they all work together?

38. A and B were employed to do a piece of work for \$60. They were to be paid in proportion to their ability to work, which was 4 to 5, and to the time each worked, which was 3 to 4. How much did each receive?

39. A boy's age now is one fifth of his father's. In six years it will be one-third of his father's present age. How old is the boy?

40. A man having lost one fifth of his capital is worth as much as another man who has gained three twentieths of his capital. The second man's capital was originally \$9000. What was the first man's capital?

41. I bought 360 gallons of wine at \$2.60 a gallon; paid for carriage \$17.20, and for duties \$86.50. If .15 of it be lost by leakage, at what price per gallon must the remainder be sold to gain \$50 on the whole transaction?

42. A person buys four houses; for the second he gives half as much again as for the first; for the third, half as much again as for the second; and for the fourth as much as for the first and third together. He pays in all \$24000. What is the cost of each?

43. Brown purchased  $\frac{7}{26}$  of a mill property for \$4064.55, and Smith purchased  $\frac{9}{35}$  of the same property at rate  $\frac{1}{10}$  higher. What did Smith's part cost him?

44. Out of a sum of \$4000, \$217 are allotted to charity, and of the remainder A gets  $\frac{1}{10}$  more than B, and B  $\frac{1}{10}$  more than C. What are the shares of each?

45. Gun metal is composed of 11 parts of copper to 2 parts of tin. How much tin must be added to 2607 lb. of copper to make gun metal?

46. What is the hour when  $\frac{2}{3}$  of the time past noon is  $\frac{2}{5}$  of the time till midnight?

47. A vessel contains 150 gallons of wine; 50 gallons are drawn and the vessel filled with water. If this operation be performed five times in all, how much wine will the mixture contain?

48. Find the G. C. M. and L. C. M. of

$$\frac{2}{3}, \frac{4}{9}, \frac{10}{63}, \frac{10}{21} \text{ and } \frac{20}{81}.$$

49. Find, correct to 6 decimal places, the value of

$$\frac{1}{8} + \frac{1}{8^2} + \frac{1}{8^3} + \text{etc.}$$

50. A bankrupt's assets are found to be  $\frac{3}{4}$  of his liabilities, but on examination  $\frac{1}{3}$  of his assets prove to be worth only 50 cents on the dollar. How many cents on the dollar can he pay to his creditors?

51. A man bought a horse and carriage for \$280; one third of the cost of the carriage was equal to one fifth of the cost of the horse. Find the cost of each.

52. Find two numbers such that their difference is 18, and that  $\frac{2}{3}$  of one will be equal to  $\frac{4}{15}$  of the other.

53. A and B rent a pasture for \$42.50. A puts on 2 cows and 3 horses, and B puts on 3 cows and 2 horses. If a horse eats  $\frac{1}{2}$  as much again as a cow in the same time, and the cows are on the pasture  $\frac{1}{3}$  longer than the horses, how much should each of the men pay?

54. In an election  $\frac{1}{16}$  of the electors refused to vote; the elected candidate received  $\frac{19}{40}$  of the whole vote in the constituency and had a majority of 50 over his opponent. How many votes were polled for each candidate?

55. If copper weighs 500 lb., lead 600 lb., tin 480 lb., respectively, to the cubic foot, find the weight of a cubic foot of metal composed of equal weights of copper, lead and tin.

56. The average of seven numbers is 26.98. The average of the first two is 34.5 and of the next three 19.3. Find the average of the remaining two.

57. A grocer receives \$9.60 for a bill of goods weighed on scales that gave only  $15\frac{1}{4}$  ounces to the pound. How much did he cheat his customer?

## Compound Quantities

---

Quantities of the same kind, but consisting of units of different denominations, are called **Compound Quantities**.

Thus 2 yd. 1 ft. 7 in. is a compound quantity.

Here there are three units of measurement, a yard, a foot and an inch. These units are of different denominations, but they are all of the same kind, because each represents a definite length.

When the units are of the same kind, and the relations between them are known, the compound quantity may be reduced to a **Simple Quantity** expressed in terms of a single unit.

Thus: 2 yd. 1 ft. 7 in.

$$\frac{3}{7} = \text{number of feet in 2 yd. 1 ft.}$$

$$\frac{12}{91} = \text{number of inches in 2 yd. 1 ft. 7 in.}$$

Here the reasoning is as follows:

$$1 \text{ yd.} = 3 \text{ ft.}$$

$$\therefore 2 \text{ yd.} = 3 \text{ ft.} \times 2 = 6 \text{ ft.}$$

$$\therefore 2 \text{ yd. 1 ft.} = 7 \text{ ft.}$$

$$1 \text{ ft.} = 12 \text{ in.}$$

$$\therefore 7 \text{ ft.} = 12 \text{ in.} \times 7 = 84 \text{ in.}$$

$$\therefore 7 \text{ ft. 7 in.} = 91 \text{ in.}$$

This operation is frequently called **Reduction Descending**, because the change is made from a larger to a smaller unit.

When the change is from a smaller to a larger unit, the operation is called **Reduction Ascending**.

Reduce 151546 in. to higher denominations.

12	151546 in.
3	12628 ft. 10 in.
	4209 yd. 1 ft.
	2
11	8418 half yd.
320	765 rd. 3 half yd. = 765 rd. 1 yd. 1 ft. 6 in.
	2 mi. 125 rd.

$$\begin{aligned} \therefore 151546 \text{ in.} &= 2 \text{ mi. } 125 \text{ rd. } 1 \text{ yd. } 1 \text{ ft. } 10 \text{ in.} \} \\ &\qquad\qquad\qquad 1 \text{ ft. } 6 \text{ in.} \} \\ &= 2 \text{ mi. } 125 \text{ rd. } 2 \text{ yd. } 0 \text{ ft. } 4 \text{ in.} \end{aligned}$$

### EXERCISE

1. How many linear inches are there in a linear rod? How many in a mile?
2. Reduce 30 weeks 6 days 12 hours to minutes.
3. Find the number of minutes in the year 1900.
4. How many cubic yards are there in a cubic mile?
5. How many days are there from Mar. 5th to Sept. 20th of the same year.
6. Reduce £100 to pence.
7. Reduce 10,000 farthings to £ s., etc.
8. If a linear chain = 22 yards, how many square chains are there in one acre?
9. If the 24th of May falls on a Monday, on what day of the week will Christmas day of the same year fall?

10. If the sun rises at 6.22 a.m. and sets at 7.37 p.m., how many minutes of daylight are there?
11. How many tons are there in 10 million ounces?
12. A dealer buys 100 long tons of coal at \$6.75 per ton. He sells it at the same price per short ton. Find his gain. (A long ton = 2240 lb., a short ton = 2000 lb.)
13. If a franc be worth  $9\frac{1}{2}$ d., how many francs are equivalent to £86 1s.  $10\frac{1}{2}$ d.?
14. Three horses together weigh 2 tons 491 pounds. Find in pounds their average weight.
15. How many revolutions will a carriage wheel 6 ft. 2 in. in circumference make in a journey of 3 mi. 970 yd. 2 ft. 10 in.?
16. W. E. Gladstone was born on Dec. 9th, 1809, and died on May 19th, 1898. How long did he live?
17. How many bags are required to hold 111 bu. 2 pk. 4 qt. of wheat if each bag holds 2 bu. 1 pk. 4 qt.?
18. How many yards of cloth at 5s. 6d. a yard are equal in value to 55 lb. of tea at 2s. 6d. a lb.?
19. On Nov. 29th, 1899, a person had lived 10480 days. When was he born?
20. If a train travels 30 miles per hour, what is its rate in feet per second?
21. If light travels at 186000 miles per second, and the distance of the sun is 93,930,000 miles, how many minutes does it take the sun's light to reach us?
22. A person takes 99 steps in a minute, each step being 30 inches in length. How far would he walk in an hour?
23. If a dealer buys 144 lb. of an article by avoirdupois weight and sells it by troy weight, how many pounds can he sell?
24. The rails of the Great Western Railway, England, weigh  $97\frac{1}{2}$  lb. per yard. Find in tons the weight of the rails required to construct a mile of this railway.
25. If a cow gives 12 qt. 1 pt. of milk every day, and 1 lb. 8 oz. of butter can be made from 25 qt. of milk, how many lb. of butter can be made in one week from the milk of 16 cows?

26. If 3 lb. of wheat make 2 lb. of flour, how many barrels of flour can be made from 343 bushels of wheat?

27. If the imperial gallon contains 277·2 cubic inches and holds 10 lb. of water, what is the error in saying that a cubic foot of water weighs 1000 oz.?

28. Find the greatest common measure of 157 da. 7 h. 4 min. 7 sec. and 243 da. 2 h. 11 min. 49 sec.

29. If a person whose income is £365 a year spends £8 16s. 3d. a week for the first 20 weeks, to what amount must he limit his daily expenditure for the remainder of the year so as to avoid being in debt at the end of it?

30. Find the number of days in each 400 consecutive years.

31. How many days will there be in the century which began on Jan. 1st, 1901?

### PROBLEMS—METRIC SYSTEM

1. Express 1000 centimetres in metres; 1000 centimetres in decimetres; 1000 centimetres in dekametres.

2 Express 1000 metres in kilometres; 1000 metres in dekametres 1000 metres in decimetres.

3. Express 12 kilometres in dekametres; 15 kilometres in metres 21 kilometres in millimetres.

4. Express 123,456,789 millimetres in decimetres; in metres; in kilometres.

5. Express 8·56 Km. in centimetres; 5·632 m. in millimetres; 12468 mm. in microns.

6. Add 14·6 m., 227 cm., 162·3 Dm., 1634 Km.; express the result in metres.

7. Express in centimetres the difference between 5·678 Km. and 1364·89 Dm.

8. Multiply 12 Km. 5 m. 8 cm. by 96.

9. Given that 64 miles is very nearly equal to 103 Km., express 1 yard in centimetres.

10. A train is running at the rate of 66 Km. per hour. How many metres does it go at this rate in one second?

11. A train running at the rate of 60 Km. per hour passes over 20 spaces between telegraph poles in one minute. Find the distance in metres between two consecutive poles.

12. Using the approximate value, 1 metre = 39.37 in., express the height of a man, 5 ft.  $10\frac{1}{2}$  in., in centimetres.

13. Using the same approximation, express in millimetres the height of a barometer which stands at 29.5 in.

14. Find the length of the two parts into which a string 10 m. long is divided, given that one half of one part and two-thirds of the other make a length of 600 cm.

15. How long will it take a man to walk from Toronto to Hamilton, a distance of 65 Km., at the rate of 80 m. per minute?

16. Taking a centimetre as  $\frac{2}{5}$  of an inch, find the number of millimetres in 1 yd.

17. How many lengths each 1 m. 5 cm., can be cut from a length of 1 Km.? Express the remainder in mm.

18. Express 1200 ares in hectares; 1200 ares in centiares; 1200 ares in dekares.

19. Find the sum of 12.64 arcs, .0468 hectares, one million milliares. Express the result in ares.

20. Express in dekares the difference between 1 Ha., and  $\frac{1}{4}$  are.

21. Multiply 7 Ha., 5 Da. 6 a. 4 da. 5 ca. by 27.

22. Express 1 square decimetre in milliares.

23. Express in square centimetres 1 are 5 da.

24. Using the approximation 1 Ha. = 2.5 acres, express 1 metre in inches.

25. How many cubic centimetres in 1 stere?

26. How many steres in one million cubic centimetres?

27. Express 1 ds. in cubic decimetres.

28. Given that 1 cubic centimetre of water weighs 1 gramme, find in kilos. the weight of one stere of earth which is 2.5 times as heavy as water.

29. Water expands  $\frac{1}{10}$  in freezing; find the weight of 1 stere of ice.

30. How many cubic decimetres in 1 cubic metre? How many litres in 1 stere?

31. Given that 1 cu. cm. of water at  $4^{\circ}\text{C}$ . weighs 1 gramme, find the weight of 1 litre of water.

32. How many litres of water at  $4^{\circ}\text{C}$ . will weigh 1000000 grammes?

33. Given that a solid loses in weight when immersed in water, an amount equivalent to the weight of the water displaced, find the volume of a solid which weighs 1000 grammes less when immersed in water at  $4^{\circ}\text{C}$ ., than when weighed in air.

34. Find approximately the weight of a kilolitre of air under certain standard conditions, given that water is 770 times as heavy as air.

35. Find the number of cubic centimetres in one litre.

36. Express the kilogramme in milligrammes.

37. Find the number of grammes in 1 tonneau.

38. Find in kilos. the weight of 10 cubic metres of water at  $4^{\circ}\text{C}$ .

39. What is the weight of 1 litre of mercury in kilos., given that mercury is 13.5 times as heavy as water?

40. A litre of sulphuric acid weighs 1840 grammes; compare the weights of equal volumes of sulphuric acid and water.

41. Given that an ounce Av.=28.35 grammes, find the number of kilos. in a ton (2000 lb.).

42. A man walks 5 Km. per hour; express this rate in cm. per second.

43. Express the chain (66 ft.) in metres.

44. Express in hectares the area of a farm which contains 250 acres.

45. The length of the Great Western Railway is 229 miles; find its length in kilometres.

46. Express 5280 feet in metres.

47. If 3 kilometres are as much under 2 miles as 5 kilometres are over 3 miles; find the length of the metre in inches.

48. How many litres of water at temperature of maximum density will weigh one tonneau?

# Powers of Numbers

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## I.

Write the following products in the exponential form, and find their values:

- |                                    |                                                                                                 |
|------------------------------------|-------------------------------------------------------------------------------------------------|
| 1. $2 \times 2$ .                  | 12. $\frac{1}{2} \times \frac{1}{2}$ .                                                          |
| 2. $3 \times 3$ .                  | 13. $\frac{1}{7} \times \frac{1}{7}$ .                                                          |
| 3. $10 \times 10$ .                | 14. $\frac{3}{11} \times \frac{3}{11}$ .                                                        |
| 4. $17 \times 17$ .                | 15. $5 \times 5 \times 5$ .                                                                     |
| 5. $20 \times 20$ .                | 16. $10 \times 10 \times 10$ .                                                                  |
| 6. $100 \times 100$ .              | 17. $6 \times 6 \times 6 \times 6$ .                                                            |
| 7. $300 \times 300$ .              | 18. $\cdot 1 \times \cdot 1 \times \cdot 1$ .                                                   |
| 8. $\cdot 1 \times \cdot 1$ .      | 19. $02 \times \cdot 02 \times \cdot 02 \times \cdot 02$ .                                      |
| 9. $\cdot 4 \times \cdot 4$ .      | 20. $\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$ . |
| 10. $\cdot 06 \times \cdot 06$ .   | 21. $1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{3}$ .                |
| 11. $\cdot 235 \times \cdot 235$ . |                                                                                                 |

## II.

Find the value of:

- $4^2, 5^2, 23^2$ .
- $\cdot 3^2, \cdot 04^2, \cdot 016^2, \cdot 518^2$ .

3.  $(\frac{1}{2})^2$ ,  $(\frac{3}{4})^2$ ,  $(\frac{11}{14})^2$ .

4.  $(.21689)^2$ ,  $(.04635)^2$ ,  $(4.623)^2$ , correct to four places of decimals.

5.  $(10)^2$ ,  $(100)^2$ ,  $(1000)^2$ .

6.  $7^3$ ,  $14^3$ ,  $163^3$ .

7.  $.1^3$ ,  $.02^3$ ,  $.0001^3$ .

8.  $(\frac{2}{3})^3$ ,  $(\frac{9}{13})^3$ ,  $(1\frac{3}{10})^3$ .

9.  $(.6932)^3$ ,  $(.02475)^3$ , correct to five places of decimals.

10.  $4^2 \times 3^3$ ,  $17^2 \times 5^3$ ,  $12^2 \times 10^3$ .

11.  $7^4$ ,  $3^5$ ,  $2^7$ .

12.  $(1.046)^9$ ,  $(1.06)^{15}$ , correct to four decimal places.

## Roots of Numbers

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1. Obtain the square roots of the following: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 144, 196, 256, 324,  $64 \times 81$ , 3600, 8100,  $16 \times 25 \times 49$ ,  $64 \times 36 \times 100$ , 640000, 81000000.

2. Obtain the cube roots of the following: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000,  $8 \times 343$ ,  $27 \times 512$ ,  $125 \times 729$ ,  $8 \times 1000$ , 125000, 512000,  $27 \times 216 \times 343$ ,  $27 \times 729 \times 1000$ , 8000000, 729000000.

3. By resolving into prime factors, obtain the square roots of the following: 36, 64, 100, 576, 729, 1024, 1296, 1764, 6400, 1089, 57600, 640000, 313600, 78400, 11025.

4. Obtain the cube roots of the following in a similar manner: 8, 27, 512, 1000, 216000, 27000, 1728, 9261, 1331, 1331000, 27000000, 42875.

5. Obtain the fifth roots of the following in a similar manner: 32, 243, 3125, 1024, 7776, 3200000, 4084101.

6. Obtain the square roots of the following in a similar manner:  $\frac{1}{4}$ ,  $\frac{1}{9}$ ,  $\frac{9}{64}$ ,  $\frac{49}{100}$ ,  $\frac{81}{10000}$ ,  $\frac{64}{729}$ ,  $\frac{100}{729}$ ,  $\frac{64}{11025}$ , .01, .04, .09, .64, 5.76, 10.24, 17.64, 110.25, 1.1025,  $\frac{448}{5103}$ .

7. Obtain the cube roots of the following in a similar manner:  $\frac{1}{8}$ ,  $\frac{1}{27}$ ,  $\frac{64}{125}$ ,  $\frac{343}{1000}$ ,  $\frac{1000}{729}$ ,  $\frac{64}{9261}$ ,  $\frac{42875}{4096}$ , .001, .008, .064, 1.728, 1.331, 9.261, 42.875, .000001, .001728, .042875.

## SQUARE ROOT

## I.

1. Multiply 35 by 35.
2. In this example, how many *units* are there in the product of 5 by 5?
3. How many *units* in the product of 3 by 5, and how often is this product found in the whole operation?
4. How many *units* in the product of 3 by 3?
5. What do we obtain if we add the results of examples 2, 3, and 4 together?
6. What is the sum?
7. If the product of 3 by 3 is subtracted from the *proper part* of this sum, what is the remainder?
8. Could any square integer greater than  $3^2$  or 9 be subtracted in a similar way from 1225?
9. How then can the tens digit of the square root of 1225 be found?
10. How are the digits 3 and 5 combined to form the remainder in example 7?
11. If twice 3 tens is divided into this remainder (since twice 3 tens  $\times$  5 forms the greater part of it); what is the quotient.
12. What should be added to the divisor twice 3 tens or 60, to make the division exact?
13. Divide 65 into the remainder of example 7, and find the remainder.
14. As this remainder is 0, what has been subtracted altogether from 1225?

15. What is the tens digit of the square root? The units digit? The whole square root?

16. How was the tens digit of the square root obtained?

17. How was the units digit of the square root *first* obtained?

18. What was added to the trial divisor of example 11 to give a divisor which would divide exactly?

19. If the number had been 1230, after going through the same operation to find its square root, what would the remainder be?

20. When we obtain 5 for a remainder, what has been subtracted from 1230?

21. What is the number subtracted from 1230 the square of?

22. Extract the square roots of 625, 631, 961, 970, and give the remainder in each case.

23. In an example in *long* division, what is used as a trial divisor to obtain the different digits of the quotient?

24. Does the trial divisor always indicate accurately the number of times the whole divisor will be contained in the dividend? Why?

25. In a case where it does not give the right digit, what is your next step?

26. Should the trial divisor in example 11 always give a quotient which will hold when the complete divisor is used?

27. When it does not, how should you proceed?

## II.

1. What is the square of 3?
2. What must a number be multiplied by to increase it ten fold?
3. If 3 is increased ten fold, how many fold must the square of 3 be increased to obtain the square of 30?
4. If 30 is increased ten fold, how many fold must the square of 30 be increased to obtain the square of 300?
5. If any number is increased ten fold, how many fold must the square of the number be increased to obtain the square of the new number?
6. How many digits in the product of a units digit by a units digit? of a tens by a tens? of a hundreds by a hundreds?
7. In obtaining any product, what is the digit of *lowest denomination* which is affected by the product of a units digit by a units digit? of a tens by a tens? of a hundreds by a hundreds?
8. What part of 568516 can be omitted, if we wish to obtain only the hundreds digit in the square root?
9. What part can be omitted if we wish to obtain both the hundreds and tens digits?
10. Find the hundreds and tens digits in the square root of this number. (Use only the digits of the number which are required.)
11. How many tens in 7 hundreds + 5 tens?
12. When 75 tens squared is subtracted from 5685 hundreds, what is the remainder?

13. When it is subtracted from 568516, what is the remainder?

14. When we have obtained this remainder and the total number of tens in the square root, what is the trial divisor for finding the units digit?

15. What is the complete divisor?

16. What is the remainder after this division?

17. What is the square root of 568516?

18. Every time we found a digit in the square root, how many digits of the original number did we use?

19. Draw vertical lines between the digits of the number 840889, so as to indicate the digits which enter together into the operation of finding the different digits of its square root; and find its square root.

20. Find the square roots of 687241, 151321, 45369, 87025.

### III.

1. What is the square of 1, .1, .01, .001?

2. What is the square root of 1, .01, .0001, .000001?

3. How many digits must we have in a decimal to give 1 decimal place in the square root? to give 2 places? to give 3 places?

4. For every digit in the square root, how many must we have in the decimal?

5. From what place should we begin to mark off the decimal in periods containing two digits each?

6. Extract the square roots of .09, .0049, .000081, .000004, 1.21, 1.44, 50.41, 26.01, 6772.41, 4.5369, 297.6423, 354.5, 2649.5, .4, to 4 places when possible.

## CUBE ROOT

When a number is a perfect cube, its cube root may be found by factoring as shown in the problems on page 76.

There is a formal method of finding the cube root of any number, similar to the method for finding the square root. It requires, however, a great amount of computation and is therefore of little practical value. The cube root of any number can be easily found, to any required degree of approximation, by the use of logarithms. For illustrations of this method of finding the cube root or higher roots of numbers, the student is referred to tables of logarithms.

## EXERCISE

- Find, by factoring, the cube roots of 9261, 32768 and 373248.
- Find the square roots of  $9\frac{67}{121}$  and  $37\frac{212}{841}$ .
- Find the square roots of .027 and 4.694.
- Find the cube roots of  $\frac{64}{729}$ ,  $\frac{2744}{9261}$  and  $3\frac{3}{8}$ .
- Find the value of  $8 \div \sqrt{8}$  correct to 4 decimal places.
- Find the value of  $\frac{1}{\sqrt{2}}$  to 4 decimal places, and show that  $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ .
- Which is the greater,  $2\sqrt{3}$  or  $3\sqrt{2}$ ? Find the difference between them to 3 decimal places.
- Find the least multipliers which will make 12825 and 29040 each a perfect square.
- Find the least multiplier which will make 90250 a perfect cube.
- Find the value of  $\sqrt{5} + \sqrt{3}$  to 3 decimal places.
- Show that  $103 \times 104 \times 105 \times 106 + 1$  is a perfect square.

12. If the units digit of a number is 7, what is the units digit of its square? Of its cube?

13. Show that no square number can have 2, 3, 7, or 8 as its units digit.

14. Show that the square root of the sum of two numbers is not equal to the sum of their square roots.

15. The product of two consecutive integers is 112560. Find them.

16. I buy a number of yards of cloth for \$56.25. The cost in cents of each yard is the same as the number of yards bought. Find the cost per yard.

17. A body of soldiers in columns form 567 ranks, 7 abreast. If they were drawn up in a solid square, how many would there be on each side?

18. The population of a city in 1881 was 136900, and in 1901 it was 148225. If the rate of increase for each decade was the same, find the population in 1891.

19. Find the sum of all the numbers in the multiplication table up to 12 times by showing that it is equal to

$$(1+2+3+\dots+12)^2.$$

20. The product of two numbers is 43923. One of them is 3 times the other. Find the numbers.

21. The ratio of two numbers is 6 to 7. Their product is 1512. Find the numbers.

22. The product of 3 numbers is 3072. The second is twice the first and the third is three times the first. Find the numbers.

# Mensuration

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## RECTANGLE

### I.

1. Place 3 units (square feet) in the form of a rectangle.
2. What is the length of this rectangle?
3. What is the width of this rectangle?
4. Place a second rectangle, equal in all respects to the former, with a side contiguous throughout to a side of the former, and in the same plane.
5. What figure do the two rectangles form?
6. What is the length of this rectangle?
7. What is the width of this rectangle?
8. What is its area in terms of the area of the first rectangle?
9. What is its area in square feet?
10. Place a third rectangle, equal to either of the others, alongside the second, in a manner similar to that in example 4.
11. What is the length of this figure?
12. What is the width of this figure?
13. What is its area in terms of the area of the first rectangle?
14. What is its area in square feet?

15. What is its length in yards?
16. What is its width in yards?
17. What name do you give the figure?
18. How many square feet in a square yard?

## II.

1. Place 4 square units in the form of a square.
2. What is the length of a side of this square?
3. If this square unit is one-half foot long and one-half foot wide, what is the length of a side of the whole square?
4. How many of these units in a square foot?
5. Make a rectangle whose length is 8 and width 5 of the linear units corresponding to the square unit in example 3.
6. What is the area of this rectangle in those square units?
7. What is the area of this rectangle in square feet?
8. What is the area of a rectangle 4 feet in length and  $2\frac{1}{2}$  feet in width?

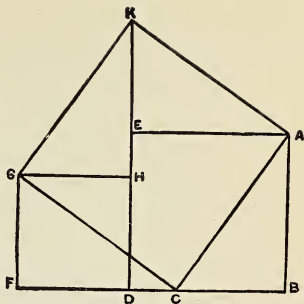
Find the areas of the following rectangles:

9. 7 rods in length and  $5\frac{1}{2}$  rods in width.
10.  $3\frac{1}{4}$  yd. in length and  $2\frac{3}{4}$  yd. in width.
11. 7 yd. 2 ft. 4 in. in length and 2 ft. 7 in. in width.
12. Prove that  $30\frac{1}{4}$  square yards are equal to 1 square rod, being granted that  $5\frac{1}{2}$  linear yards are equal to 1 linear rod.

## TRIANGLE

Let  $ABC$  be a right angled triangle, with the angle  $ABC$  a right angle.

Let  $ABDE$  be a square described on  $AB$ , and  $DFGH$  a square equal to one described on  $BC$ , and let  $BD$  and  $FD$  be coterminous at  $D$ , and in the same straight line.



Join  $GC$ .

Remove the triangle  $GFC$  and place it in the position of  $GHK$ .

Also remove the triangle  $ABC$  and place it as  $AEK$ .

The figure  $ACGK$ , which is the square on  $CA$ , is composed of the same parts as the two squares  $ABDE$  and  $DFGH$ .

$\therefore$  the squares described on  $AB$  and  $BC$  are equal to the square described on  $CA$ .

NOTE.—If a piece of paper shaped like  $ABFGHE$  be cut along the lines  $AC$  and  $GC$ , then the parts can be put together in the shape of the figure  $ACGK$ .

Find the length of the hypotenuse in each of the following right angled triangles, given:

- |    |           |        |     |        |
|----|-----------|--------|-----|--------|
| 1. | The sides | 5 ft.  | and | 12 ft. |
| 2. | "         | 9 ft.  | "   | 40 ft. |
| 3. | "         | 13 ft. | "   | 84 ft. |
| 4. | "         | 8 yd.  | "   | 15 yd. |
| 5. | "         | 14 in. | "   | 2 ft.  |

6. The sides 2 ft. 2 in. and 4 ft. 6 in.

7. " 7.64 ft. " 2.9 ft.

Find the third side in each of the following right angled triangles, given:

8. The hypotenuse 10 ft. one side 8 ft.

9. " " 41 ft. " 9 ft.

10. " " 113 ft. " 15 ft.

11. " " 14 yd. " 6 yd.

12. " " 4 yd. 2 ft. 7 in. " 3 yd. 1 ft. 9 in.

13. " " 9.72 ft. " 8.6 ft.

1. Draw an acute angled triangle having an altitude 10 units, and a base 7 units in length.

2. On the base describe a rectangle having the same altitude and towards the same parts as the triangle.

3. Into how many parts does the altitude divide the triangle?

4. Into how many parts does the altitude divide the rectangle?

5. What is the area of either part of the rectangle in terms of the corresponding part of the triangle?

6. What is the area of the whole rectangle in terms of the whole triangle?

7. What is the area of the whole triangle in terms of the whole rectangle?

8. What is the area of the rectangle in square units? And what the area of the triangle?

Find the areas of the following triangles:

9. Base 4 ft. and altitude 3 ft.

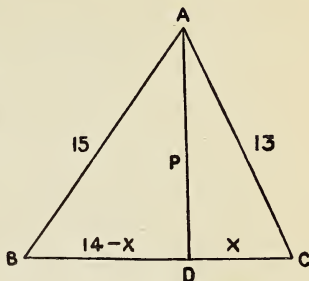
10. Base 2.79 yd. and altitude 8 ft.

11. Base 2.59 in. and altitude 4.68 ft.

Example—The sides of a triangle are 13, 14 and 15 units in length; find the length of the perpendicular dropped on the side 14, from the opposite angle. Find also the area of the triangle.

In the diagram let AD represent the perpendicular and let it contain  $p$  units. Also, let DC contain  $x$  units.

$\therefore$  DB contains  $14 - x$  units.



In the triangle ADC,  $p^2 + x^2 = 13^2$  (1)

In the triangle ADB,  $p^2 + (14 - x)^2 = 15^2$  (2)

Subtract (1) from (2) and we get  $(14 - x)^2 - x^2 = 15^2 - 13^2$

$$\therefore 196 - 28x = 56$$

$$\therefore 28x = 140$$

$$\therefore x = \frac{140}{28} = 5.$$

Substitute this value of  $x$  in (1) and  $p^2 + 5^2 = 13^2$

$$\therefore p^2 = 144$$

$$\therefore p = \sqrt{144} = 12.$$

But the measure of the area of the triangle ABC is  $\frac{1}{2} AD \times BC$ ,

$\therefore$  the area is  $(\frac{1}{2} \times 12 \times 14)$  square units = 84 square units.

When  $a$ ,  $b$  and  $c$  are the measures of the sides of a triangle, the measure of the area is  $\sqrt{s(s-a)(s-b)(s-c)}$  where  $2s = a + b + c$ .

This result may be obtained in a similar manner, but to do so requires a considerable knowledge of algebra. The formula is an important one, and should be remembered.

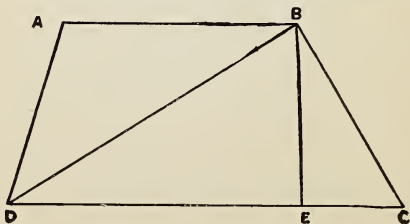
### EXERCISE

Find the segments of the base of a triangle into which the perpendicular dropped from the opposite angle divides it; also the length of the perpendicular, the base being given last in each case.

1. Sides are 5, 12, 13 units.
2. Sides are 9, 40, 41 ft.
3. Sides are 26, 168, 170 yd.
4. Sides are 5, 9, 8 ft.
5. Sides are 6 ft., 7 ft., 49 in.
6. Find the areas of the triangles in the preceding examples, (a) from the base and the perpendicular which has been found, (b) by substituting in the general formula.
7. Show how to find the length of the perpendicular in any of these cases by first finding the area from the formula.

### TRAPEZIUM

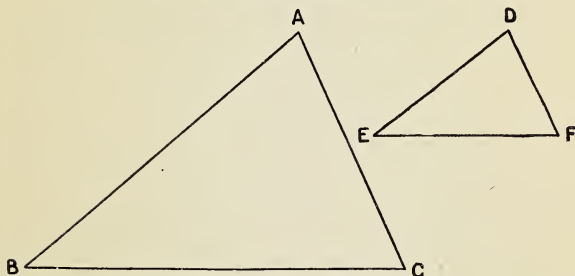
Let  $ABCD$  be a trapezium, having  $AB$  and  $CD$  parallel. Let  $BE$  be the perpendicular drawn from  $B$  to  $DC$ . Let the lengths of  $AB$ ,  $DC$  and  $BE$  be  $a$ ,  $b$ , and  $h$  units respectively.



Then the measure of the area of the triangle  $BDC = \frac{1}{2} bh$ ,  
and the measure of the area of the triangle  $ABD = \frac{1}{2} ah$ ,  
therefore the measure of the area of the trapezium

$$\begin{aligned} &= \frac{1}{2} ah + \frac{1}{2} bh \\ &= \frac{1}{2} h (a + b) = h \left[ \frac{a + b}{2} \right]. \end{aligned}$$

### SIMILAR TRIANGLES



If the triangles  $ABC$  and  $DEF$  are such that  $\angle A = \angle D$ ,  $\angle B = \angle E$  and  $\angle C = \angle F$ , that is, if they are equiangular, it is proven in geometry that the ratios of the corresponding sides are equal, or that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

Such triangles are said to be similar.

If  $AB = 2 DE$ ,

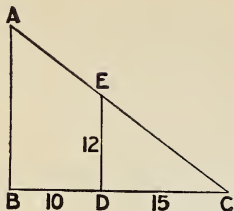
then  $BC = 2 EF$ , and  $AC = 2 DF$ .

If  $AB = 7$ ,  $BC = 8$ ,  $AC = 5$  and  $DE = 3$ ,

then since  $DE = \frac{3}{7}$  of  $AB$ ,

$\therefore EF = \frac{3}{7}$  of  $BC = \frac{3}{7}$  of  $8 = 3\frac{3}{7}$ ,

and  $DF = \frac{3}{7}$  of  $AC = \frac{3}{7}$  of  $5 = 2\frac{1}{7}$ .



In the figure, if the angles  $ABC$  and  $EDC$  are equal, then the triangles are equiangular and therefore similar.

If  $BD = 10$ ,  $DC = 15$  and  $DE = 12$

$$\text{Then } \frac{AB}{DE} = \frac{BC}{DC}$$

$$\therefore \frac{AB}{12} = \frac{25}{15}$$

$$\therefore AB = \frac{12 \times 25}{15} = 20.$$

## CIRCLE

The circumference of a circle bears a constant ratio to its diameter; this ratio is  $3\frac{1}{7}$  nearly, but closer approximations are  $3.1416$  and  $\frac{355}{113}$ , and is always denoted by the letter  $\pi$ .

Take a Canadian cent, the diameter of which is one inch. Place it flat, and measure the length of the circumference by passing a string around it. Test with larger circles, cut out of cardboard, and show that the circumference is approximately  $3\frac{1}{7}$  times the diameter.

## EXERCISE

Given  $\pi = 3\frac{1}{7}$ , find the circumference:

1. If the diameter is 4 ft. in length.
2. " "  $5\frac{3}{4}$  yd. in length.
3. " " 4 yd. 2 ft. 3 in. in length.
4. " " 7.468 ft.

Find the diameter:

5. If the circumference is 9 ft. in length.
6. " " 7 ft. 3 in. in length.
7. " " 9.46 yd. in length.

Find the circumference:

8. If the radius is 12 ft. in length.
9. " "  $4\frac{1}{3}$  yd. in length.
10. " 4 yd. 2 ft. 7 in. in length.
11. Take any circle and ascertain, by actual measurement, which is the greater, 22 times the diameter or 7 times the circumference.

## POLYGON AND CIRCLE

Take a number of equal isosceles triangles.

1. Express the sum of the areas of two of the triangles in terms of the area of one of them.
2. Express the sum of two bases in terms of one base.
3. What relation is there between the measure of the sum of the areas in example 1, and the measure of the sum of the bases in example 2?
4. Express the sum of the areas of 5 of the triangles in terms of the area of one of them.
5. Express the sum of 5 bases in terms of one base.
6. Find the relation which exists between the measures in examples 4 and 5.
7. If the triangles are of such a shape that when placed in a plane, with a common vertex, their bases will form the perimeter of a complete figure; what is this figure called?

8. If there are 20 triangles in this polygon, express the area of the polygon in terms of one triangle.

9. Express the perimeter of the polygon in terms of one base.

10. Express the measure of the area of one of these triangles in terms of the measures of its altitude and base.

11. Express the measure of the area of the polygon in terms of the measures of the altitude and the base of one triangle.

NOTE.—The base and the altitude of one triangle are respectively equal to the base and the altitude of any other.

12. Express the measure of the area of the polygon in terms of the measures of its perimeter and the altitude of one triangle.

13. What is to be noticed regarding the number of triangles required to make a complete polygon, if the bases of the triangles become very short, while the sides remain the same length?

14. What other figure does this one resemble?

15. If we round off the corners of this figure, so as just to destroy the angles, how much of the figure is removed?

16. What is the difference between the areas of the figures in examples 14 and 15.

17. What is the name of the figure in example 15?

18. What is the difference in length between the side of one of the triangles and its altitude in example 13?

19. In example 15, what is a side of a triangle called?

20. In example 15, what is the perimeter called?

21. Find the measure of the area of the figure in example 15 in terms of the measures of its radius and circumference.

22. Find the area of a circle whose radius is  $3\frac{1}{2}$  feet and circumference 22 feet.

23. Find the area of a circle whose radius is 4 feet.

NOTE—If  $r$  is the measure of the radius and  $c$  of the circumference the measure of the area is  $\frac{1}{2} cr = \frac{1}{2} \times 2\pi r \times r = \pi r^2$ .

24. Find the area of a circle whose radius is 5 yd. 2 ft.

25. “ “ “ “ 6.42 ft.

26. “ “ “ diameter is  $2\frac{3}{4}$  in.

27. “ “ circumference is 11 ft.

28. “ “ “ “ 9 ft. 4 in.

## SECTOR OF A CIRCLE

1. In a circle whose circumference is 144 in., what part is an arc of 12 in. of the whole circumference?

2. In the same circle, what part is a sector of a circle, which stands on this arc, of the whole area?

3. If the circumference of a circle is 140 ft., find the length of the arc on which a sector, which is one-fifth of the whole area, stands.

4. In a circle which is 72 ft. in circumference, find the area of the sector which stands on an arc 9 ft. in length?

5. Find the area of a sector of a circle which stands on an arc 10 ft. in length, if the radius of the circle is 7 ft.

6. Find the area of the sector of a circle which stands on an arc equal in length to the radius of the circle; given the radius 3 ft.

## CYLINDER

1. Take a rectangular sheet of paper, bring the two ends into contact, and have each side in the form of a circumference of a circle.

2. If the space inclosed were a solid mass, what would it be called?

3. What does an end of the paper represent on this mass?

4. What does a side represent?

5. Find the area of the surface of the paper required to inclose this solid, if the end of the paper is 8 inches and the side 14 inches in length.

6. Find the area of the curved surface of a cylinder the height of which is 7 ft., and the circumference of the base 12 ft.

7. The radius of the circular base of a cylinder is  $3\frac{1}{2}$  ft. and the height 3 ft.; find the area of the curved surface.

8. The curved surface of a cylinder contains 24 sq. ft. and the height is 4 ft.; find the circumference of the base.

9. In the preceding example, find the radius of the base.

10. If the radius of the base of a cylinder is 3 ft. and the curved surface contains 49 sq. ft.; find the height.

## CONE

1. Take a piece of paper in the form of a sector of a circle, and bring the bounding radii into contact, so that the arc will form the circumference of a circle.

2. If the space inclosed were a solid mass, what would it be called?

3. What does a bounding radius represent on this mass?

4. What does the arc represent?

5. What does the centre of the sector represent?

6. If the arc is 7 ft. and the radius 6 ft. in length, find the area of the surface of the paper required to inclose this solid.

7. Find the area of the curved surface of a cone whose slant side is 6 ft. and the circumference of the base 7 ft.

8. Given the slant side of a cone 9 ft. and the circumference of the base 12 ft., find the area of the curved surface.

9. Given the slant side of a cone 11 in. and the radius of the circular base 7 in., find the area of the curved surface.

10. Given the area of the curved surface of a cone 49 sq. in. and the slant side 7 in., find the circumference of the base.

11. In the preceding example, find the radius of the base.

12. Given the area of the curved surface 77 sq. in., and the radius of the base 2 in., find the slant side.

13. Given the slant side of a cone 5 ft., and the radius of the base 3 ft., find the perpendicular height.

14. Given the slant side 7 ft., and the radius of the base 4 ft., find the perpendicular height.

15. Given the slant side 9 in., and the circumference of the base 14 in., find the perpendicular height.

16. Given the perpendicular height 4 ft., and the radius of the base 3 ft., find the slant side.

17. Given the perpendicular height 11 in., and the circumference of the base 21 in., find the slant side.

18. Given the perpendicular height 12 in., and the radius of the base 5 in., find the area of curved surface.

19. Given the perpendicular height 13 in., and the radius of the base 6 in., find the area of the curved surface.

20. Given the perpendicular height 8 ft., and the circumference of the base 28 ft., find the area of the curved surface.

21. Given the perpendicular height 12 in., and the circumference of the base 19 in., find the area of the curved surface.

## SPHERE

Take any hemisphere or half-sphere and make a cylinder whose diameter and height are respectively equal to those of the hemisphere. If the hemisphere and the cylinder be wound with cord, it will be found that the quantities required to cover the curved surfaces of the two solids are equal.

Let  $r$  be the measure of the radius of the hemisphere.

Then the measure of the area of the curved surface of a cylinder whose diameter is  $2r$  and height  $r$  is

$$2\pi r \times r = 2\pi r^2.$$

Then the measure of the surface of a hemisphere whose radius is  $r$  is  $2\pi r^2$ .

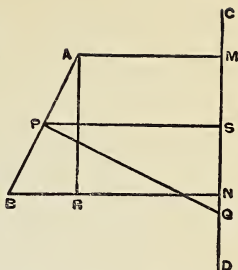
Therefore the measure of the surface of a sphere whose radius is  $r$  is  $4\pi r^2$ .

Let AB and CD represent two straight lines, and P the middle point of AB.

Draw AM, PS, BN perpendicular to CD.

Draw PQ perpendicular to AB and AR perpendicular to BN.

Revolve AB around CD as an axis.



The measure of the area of the belt which AB describes is clearly equal to the product of the measures of AB and the circumference of the circle which P describes.

$\therefore$  the measure of the area of the belt is  $2\pi \times AB \times PS$ , where AB and PS stand for the measures of the lengths of these lines.

The triangles ARB and PSQ are similar.

$$\therefore \frac{PS}{AR} = \frac{PQ}{AB}$$

$$\therefore PS \cdot AB = PQ \cdot AR, \text{ and } AR = MN.$$

$\therefore$  the measure of the area of the belt is  $2\pi \cdot PQ \cdot MN$ . (MN is called the projection of AB on CD.)

1. Take half of the perimeter of a regular polygon of an even number of sides, and draw the diagonal joining its extremities.

2. Where do the perpendiculars from the middle points of the sides cut this diagonal?

3. Compare the lengths of these perpendiculars.

4. Revolve the half polygon in example 1 around the diagonal.

5. What factors in the measures of the belts in example 4 are equal?

6. What factors are unequal?

7. What is the sum of the unequal factors?

8. What is the measure of the sum of the areas of the belts in example 4, in terms of the measures of the perpendicular from the middle point of one of the sides and the diagonal of the polygon?

9. If the sides of the polygon in example 1 become very short, and increase in number, while the diagonal remains constant, what does the solid formed by its revolution resemble?

10. If the edges are smoothed off, what is the solid called?

11. What does the perpendicular from the middle point of any one of the sides in example 1 become in example 10?

12. What is the measure of the area of the surface of a sphere in terms of the measures of its radius and diameter?

13. What is the measure of the area of a sphere in terms of the measure of its radius?

14. Find the area of the surface of a sphere whose diameter is 7 ft.

15. Find the areas of the surfaces of the spheres, the radii of which are respectively 7 ft. 4 in.; 9.265 in.; 4 yd. 2.56 ft.

16. Find the radii of the spheres, the surfaces of which are respectively 121 sq. ft.;  $5\frac{1}{2}$  sq. ft.; 4 sq. yd., 3 sq. ft.

**RECTANGULAR SOLID**

1. Place 5 units (cubic feet) in the form of a rectangular solid.
2. What is the length of this solid? What is the width?
3. Place an equal rectangular solid alongside of this one, so as to form with it a single rectangular solid.
4. What is the volume of the new solid, which is formed, in terms of the first one?
5. What is its volume in cubic feet?
6. Take 4 equal rectangular solids like the one in example 1, and place them on a plane so as to form a single rectangular solid.
7. What is the volume of this solid in terms of one of them?
8. What is the volume in cubic feet?
9. Place squarely on top of this solid another equal to it in all respects.
10. What is the new solid called?
11. What is its length, width, height?
12. What is its volume in terms of the solid in example 6?
13. What is its volume in terms of the solid in example 1?
14. What is its volume in cubic feet?
15. Place 8 equal cubes, edge  $\frac{1}{2}$  ft., in the form of a single cube.
16. What is the name of the cube so formed?

17. Find the volume of a cube whose edges are 3 ft. each.

18. What is the name of such a cube?

19. How many cubic feet in a cubic yard?

20. Find the volumes of the rectangular solids of the following dimensions: 3, 4, 5 ft. respectively;  $2\frac{1}{2}$ ,  $3\frac{1}{4}$ , 8 yd. respectively;  $2\frac{1}{4}$  yd., 4 ft., 13 in. respectively; 3·46, 5·9, 8 ft. respectively.

21. If the volume of a rectangular solid is 1728 cubic inches, and the height is 12 inches, find the area of the base.

22. If the base in example 21 be a square, find a side of it.

23. If the volume is 864 cu. ft., and the area of the base is 108 sq. ft., find the height.

24. Given the volume 725 cu. yd., the height  $7\frac{1}{2}$  yd., the length of the base  $20\frac{1}{4}$  yd., find the width of the base.

25. Given the volume 847 cu. yd., the height  $25\frac{1}{2}$  ft., one side of the base 56 inches, find the other side.

## PRISM

1. What is the volume of a solid 1 foot in height, whose base is a polygon containing 9 square feet?

2. Place another solid equal in all respects to the former exactly on top of it, so that the base of the second solid will coincide throughout with the top of the former.

3. What is the name of the new solid formed?

4. What is its volume in terms of the volume of the first?

5. What is its height?
6. What is the volume in cubic feet?
7. Place 8 solids, each identically equal to the first, on top of one another in a manner similar to that in example 2.
8. What is the new solid called?
9. What is its volume in terms of the volume of one of them?
10. What is its height?
11. What is the volume in cubic feet?
12. Find the volume of a prism 10 feet in height whose base contains 14 square feet.
13. Find the volume of a prism 3 inches in height, whose base is a square, with an edge 2 inches in length.
14. The sides of the rectangular base of a prism are 4 and 5 feet respectively, and the height is 7 feet; find its volume.
15. The sides of the base of a triangular prism are 3, 4 and 5 feet respectively, and the height is 6 feet; find its volume.
16. The sides of the base of a triangular prism are 7, 8 and 9 feet respectively, and the height is 11 feet; find its volume.

## CYLINDER

1. If the base of a prism is a polygon of a great number of sides, but each side very short, what other solid does this prism resemble?

2. If the sharp corners, along the length of the prism, are smoothed off a very little, what is the name of the new solid formed?

3. How much of the prism has been removed by smoothing off the corners?

4. What is the difference between the volumes of the solids in examples 1 and 2?

5. What is the measure of the volume of a cylinder in terms of the measures of the area of the base and the height?

6. Find the volume of a cylinder 9 feet in height, and standing on a base whose area contains 12 square feet.

7. The radius of the base of a cylinder is 4 inches in length and the height 7 inches; find the volume.

8. The circumference of the base of a cylinder is 14 feet and the height 8 feet; find the volume.

9. Given the volume 144 cubic feet and the height 9 feet, find the area of the base.

10. In the preceding example, find the radius of the base.

11. Given the volume 108 cubic inches and the area of the base 16 square inches, find the height.

12. Given the volume 98 cubic inches and the radius of the base 4 inches, find the height.

13. Given the volume 396 cubic feet and the circumference of the base 35 feet, find the height.

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The following experimental methods may be used for finding the volumes of the cone, pyramid and sphere:

**Cone.**—Construct with paper a cone and a cylinder

having equal bases and heights. Fill the cone with sand and empty the contents into the cylinder and thus show that the volume of the cylinder is three times the volume of the cone.

The measure of the volume of a cylinder has been shown to be the product of the measures of the base and height.

Therefore the measure of the volume of a cone is one-third of the product of the measures of the base and height.

**Pyramid.**—Perform the same operation with a prism and pyramid, having equal bases and heights and show that the same result will follow.

**Sphere.**—Take any hemisphere and fold a heavy sheet of paper around it, and thus make a cylinder whose internal diameter is equal to the diameter of the hemisphere. Make the height of the cylinder equal to the height of the hemisphere. Fill with sand the remainder of the cylinder. If the hemisphere is now removed, it will be found that the sand will fill one-third of the cylinder.

Therefore the volume of a hemisphere is  $\frac{2}{3}$  of the volume of a cylinder with equal base and height.

Therefore the volume of a sphere is  $\frac{2}{3}$  of the volume of a cylinder of which the height and the diameter of the base are each equal to the diameter of the sphere.

If the measure of the radius of the sphere is  $r$ , then the measure of the volume of a cylinder whose height is  $2r$  and the radius of whose base is  $r$  is  $\pi r^2 \times 2r = 2\pi r^3$ .

Therefore the measure of the volume of a sphere whose radius is  $r$  is

$$\frac{2}{3} \times 2\pi r^3 = \frac{4}{3}\pi r^3.$$

## PYRAMID

If the bases of two triangular pyramids are equal in area, and the pyramids have equal altitudes, it is proven in geometry that they have equal volumes.

1. Divide a triangular prism into 3 triangular pyramids.
2. Compare their volumes.
3. What is the volume of the whole prism in terms of the volume of one of these pyramids.
4. What is the volume of one of the pyramids in terms of the volume of the whole prism?
5. What is the measure of the volume of the prism in terms of the measures of its base and altitude?
6. What is the measure of the volume of one of the pyramids in terms of the measures of the base and altitude of the prism?
7. What is the altitude of the first pyramid which was taken from the prism?
8. What is its base in terms of the base of the prism?
9. What is the measure of the volume of this pyramid in terms of the measures of its base and altitude?
10. What is the measure of the volume of any triangular pyramid in terms of the measures of its base and altitude?
11. If we have a number of equal triangular pyramids of such a shape that when resting on their bases they have a common apex, and one edge common to all of them, what complete solid do they form?

12. What is the volume of this whole pyramid in terms of the volume of one of the triangular pyramids, if it takes 6 triangular pyramids to form it?

13. What is the area of the base of the whole pyramid in terms of the base of one of the triangular pyramids?

14. What is its altitude?

15. What is the measure of the volume of the whole solid in terms of the measures of its base and altitude?

16. If its base contains 20 square feet, and its altitude is 9 feet, what is the volume?

17. Find the volume of a pyramid 6 feet in height which stands on a square base, whose edge is 4 feet.

18. The volume of a pyramid is 72 cubic inches, and its height is 12 inches; what is the area of the base?

19. The volume of a pyramid which stands on a square base is 96 cubic inches, and its height is 14 inches; find a side of the base.

20. A pyramid which stands on a rectangular base 5 feet in length and 4 feet in width, is 7 feet high; find its volume.

21. The sides of the triangular base are 7, 9 and 10 feet respectively, and the height of the pyramid is  $4\frac{1}{2}$  yards; find the volume.

## CONE

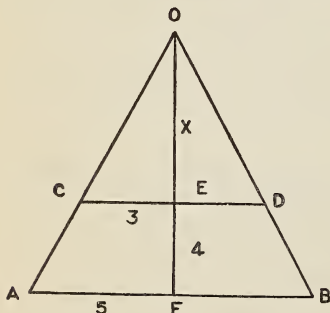
1. Take a pyramid whose base is a polygon of a very great number of sides, and each side very short.

2. What does the perimeter of the base resemble?

3. Smooth off the slant edges of this pyramid.

4. What is it now called?
5. What is the difference between the volumes in examples 2 and 3?
6. Find the volume of a cone, given its altitude 9 ft. and the area of the base 27 sq. ft.
7. Find the volume of a right circular cone 8 ft. high, the radius of whose base is 7 ft.
8. Given the volume of a right circular cone 77 cu. in. and the height 6 in., what is the area of the base?
9. In the preceding example, what is the radius of the base?
10. In a right circular cone the volume is 47 cu. yd., and height 4 yd.; find the radius of the base.

### FRUSTUM OF A CONE OR PYRAMID



Let  $OAB$  be the section of a right circular cone, made by a plane through the vertex and perpendicular, to the base.

Suppose a cone, of which  $OCD$  is a perpendicular section, to be cut from it.

The part of the cone which remains, of which  $ABDC$  is a section, is called a frustum.

Suppose  $AB=10$ ,  $CD=6$ , and  $EF=4$  units respectively, it is required to find the volume of the frustum.

Let  $OE = x$  units.

Then from the similar triangles  $OCE$  and  $OAF$ ,

$$\frac{OE}{OF} = \frac{CE}{AF}$$

$$\therefore \frac{x}{x+4} = \frac{3}{5}$$

$$\therefore x = 6$$

$\therefore$  the measure of the volume of the whole cone is  $\frac{1}{3}\pi \cdot 5^2 \times 10$   
 $= \frac{250}{3}\pi$ ,

and the measure of the volume of the small cone is  $\frac{1}{3}\pi \cdot 3^2 \times 6$   
 $= 18\pi$ ,

$\therefore$  the measure of the volume of the frustum

$$= \frac{250\pi}{3} - 18\pi = \frac{196}{3}\pi.$$

If the radius of the base of the frustum is  $R$ , the radius of the top  $r$  and the height  $h$  units respectively, then the height,  $x$ , of the small cone may be found from the equation,

$$\frac{x}{x+h} = \frac{r}{R}$$

$$\therefore x = \frac{hr}{R-r},$$

$$\therefore x+h = \frac{hR}{R-r},$$

$\therefore$  the measure of the volume of the whole cone  $= \frac{1}{3}\pi R^2 \cdot \frac{hR}{R-r}$ ,

and the measure of the volume of the small cone  $= \frac{1}{3}\pi r^2 \cdot \frac{hr}{R-r}$ ,

$\therefore$  the measure of the volume of the frustum  $= \frac{1}{3}\pi h \cdot \frac{R^3 - r^3}{R-r}$   
 $= \frac{1}{3}\pi h (R^2 + Rr + r^2).$

If  $A$  and  $a$  are the measures of the areas of the base and top of the frustum,

then  $A = \pi R^2$ , and  $a = \pi r^2$ ,

$\therefore$  the volume of the frustum  $= \frac{h}{3} (A + \sqrt{Aa} + a).$

This formula may be used to find the volume of the frustum of a pyramid whose height is given, when the areas of the base and top are given or can be found.

The surface of the frustum of the cone may be obtained by finding the difference between the surfaces of the two cones.

It is more easily found, however, by supposing the surface of the frustum to be divided into a large number of small trapeziums.

The height of each trapezium is  $BD$  ( $k$ ).

The sum of all their bases is the circumference of a circle of which  $R$  is the radius, and is therefore  $2\pi R$ .

The sum of all the opposite sides is the circumference of a circle of which  $r$  is the radius, and is therefore  $2\pi r$ .

**Therefore the measure of the surface of the frustum**

$$\begin{aligned} &= \frac{1}{2} k (2\pi R + 2\pi r) \\ &= k \pi (R + r). \end{aligned}$$

## SPHERE

Take a number of equal pyramids.

1. Place two of them side by side with their apexes at the same point.

2. What is the volume of this new solid in terms of the volume of one pyramid?

3. What is the area of the two bases in terms of the area of one base?

4. If a number of them be placed so as to form a complete solid figure, with the common apex in the centre of the solid, what is the name of this solid?

5. If it takes 20 pyramids to form this polyhedron; what is its volume in terms of the volume of one pyramid?

6. What is the area of its surface in terms of the area of the base of one pyramid?

7. What relation exists between the measures in examples 5 and 6?

8. What is the measure of the volume of this solid in terms of the measures of the altitude and the base of one pyramid?

9. What is the measure of the volume of this solid in terms of the measures of the altitude of one pyramid, and the surface of the whole solid?

10. If the bases of the pyramids become very small, what is to be noticed regarding the number required to form the solid in example 4?

11. What other solid does it now resemble?

12. What is the difference between the altitude and slant edge of a pyramid like those in example 10?

13. If the edges of the surface in example 10 are smoothed off, what is the solid then called?

14. What is the difference in volume between the solids in examples 10 and 13?

15. What are the altitudes of the pyramids in example 10 termed in example 13?

16. Find the measure of the volume in example 13 in terms of the measures of its surface and radius.

17. Find the volume of a sphere, having given the radius 7 in. and the surface 616 sq. in.

18. Find the volume of a sphere whose radius is 4 in.

19. Find the radius of a sphere whose volume is 38808 cubic inches.

20. Find the volume of a sphere whose surface is 154 square feet.

## PROBLEMS IN MENSURATION

For convenience of reference the rules for obtaining the measurement of surfaces, and the volumes of solids, are here expressed generally:

1. Where  $a$  is the measure of the side and  $b$  of the end of a rectangle, the measure of its area is  $a \times b$ .

2. Where  $a$ ,  $b$  and  $c$  are the measures of the sides of a triangle, the measure of the area is

$$\sqrt{s(s-a)(s-b)(s-c)} \text{ where } 2s = a + b + c.$$

3. Where  $h$  is the measure of the height of a cylinder, and  $c$  is the measure of the circumference of its base, the measure of the area of the curved surface is  $c \times h$ .

4. Where  $a$  is the measure of the slant side of a cone, and  $c$  is the measure of the circumference of the base, the measure of the area of the surface of the cone is  $\frac{1}{2} \times a \times c$ .

5. Where  $a$ ,  $b$ , and  $c$  are the measures of the height, length, and width, respectively, of a rectangular parallelepiped, the measure of its volume is  $a \times b \times c$ .

6. Where  $h$  is the measure of the height of a cylinder or prism, and  $A$  is the measure of the area of its base, the volume is measured by  $h \times A$ .

7. Where  $h$  is the measure of the height of a pyramid or cone, and  $A$  is the measure of the area of the base, the volume is measured by  $\frac{1}{3} \times h \times A$ .

8. Where  $c$  is the measure of the circumference of a circle, and  $r$  is the measure of the radius,  $c = 2\pi r$ .

9. The measure of the area of a circle is  $\pi r^2$ .

10. Where  $a$  is the measure of the arc of a sector of a circle, the measure of the area is  $\frac{1}{2}ar$ .

11. The measure of the area of the surface of a sphere is  $4\pi r^2$ .

12. The measure of the volume of a sphere is  $\frac{4}{3}\pi r^3$ .

13. Where  $R$  and  $r$  are the measures of the radii of the base and top respectively of a frustum of a cone, and  $h$  is the measure of the height, the measure of the volume is  $\frac{1}{3}\pi h (R^2 + Rr + r^2)$ .

14. The measure of the area of a trapezium is  $\frac{1}{2} (a + b) \times h$ , where  $a$  and  $b$  are the measures of the parallel sides and  $h$  is the measure of the altitude.

1. A room is 24 feet by 20 feet; in the central part is a carpet which measures 21 feet by 17 feet. Find the cost of painting the rest of the floor at 12 cents a square yard.

2. A foot path goes up the side, and then along the end of a rectangular field, 216 yards long and 195 broad. What distance will be saved by cutting right across in the direction of the diagonal?

3. A ladder, 40 feet long, is placed so as to reach a window 24 feet high on one side of a street, and from the same spot it will reach a window 32 feet high on the other side of the street. Find the breadth of the street.

4. Find how many trees there are in a wood half a mile long and 50 rods wide, supposing on an average four trees grow on each square chain.

5. Find the number of cedar blocks required to pave a street 40 feet wide and 300 yards long, supposing that a block occupies 24 sq. inches.

6. What is the surface of a board 18 in. wide at one end, 25 in. at the other, and 16 feet long?

7. Find the expense of paving a road of the uniform breadth of 4 yards around the inside of a rectangular piece of ground, the length of which is 85 yards and breadth 56 yards, the cost of paving being 25 cents per square yard.

8. Find in feet, to three decimal places, the side of a square containing  $2\frac{3}{4}$  acres.

9. Find how many persons can stand in a room measuring 15 feet by 9 feet, supposing each person to require a space 27 inches by 18 inches.

10. A line reaching from the top of a precipice 130 feet high on the bank of a river, to the opposite side, is 380 feet long. How wide is the river?

11. The two sides of an isosceles triangle measure 65 feet each, and the base is 50 feet. What is the altitude?

12. A ladder, 30 feet long, stands upright against a wall. Find how far the bottom of the ladder must be pulled out to lower the top 6 feet.

13. Find how many square flower beds, 4 feet to the side, can be arranged in a square plot whose side is 7 feet.

14. The base of a triangular field containing 1 acre, is  $90\frac{1}{3}$  yards in length. What is the altitude?

15. The rent of a square field, at \$12 an acre, is \$132.24. Find the cost of putting a fence around it at 35 cents a yard.

16. A piece of cloth, five times as long as it is broad, costs £38. Find its dimensions if the price is 9s. 6d. per square yard.

17. Find the cost of paving with asphalt a walk 6 feet wide, around the outside of a block 30 yards wide and 50 yards long, at \$1.50 per square yard.

18. Find the width of a rectangular field which contains 12 acres, and which measures 24 chains in length.

19. The sides of a triangle measure 14, 12 and 9 chains respectively. What is its area in square yards?

20. A field is ten thousand times as large as the plan which has been made of it. Find what length in the plan will represent a length of 24 yards in the field.

21. Find, in inches, the side of the greatest square stick of timber which can be cut from a tree whose circumference is 12 feet.

22. A rectangular court measures 21 ft. 6 in. by 13 ft. 4 in. Find the cost of paving it at 6 pence a square foot.

23. A room is 24 feet long, 18 feet broad, and 7 feet high. What length of string will reach from any corner of the floor to the farthest corner of the ceiling?

24. The sides of a triangle are 21, 43 and 35 feet respectively. Find its area.

25. The sides of a quadrilateral field are 20, 30, 25 and 32 chains respectively, and one diagonal is 40 chains. Find its area in square yards, obtaining two different results.

26. The side of a square is 8 feet. Find the area of a circle described about it.

27. The radius of a circle is 2 feet. Find the whole perimeter of its semicircle.

28. The whole perimeter of a semicircle is 80 inches. Find its radius.

29. A road runs around a circular shrubbery; the outer circumference is 560 feet, and the inner 420 feet. Find the breadth of the road.

30. What is the diameter of a wheel which turns round 1000 times in travelling a mile?

31. Find the width of a circular path containing 120 sq. yards, which surrounds a circular pond whose circumference is 220 yards.

32. Find the dimensions of a rectangle containing 240 sq. ft., if its length is to its width as 3 to 2.

33. Which requires more fence, a circular field 15 rods in diameter, or a square one whose side is 14 rods?

34. Find the cost of paving a road of the uniform breadth of 4 yards around the inside of a rectangular piece of ground, the length of which is 85 yards and the breadth 50 yards, the cost of paving being 20 cents a square yard.

35. A box is without a lid ; if the external length is 3 ft., width 2 ft., depth 1 ft. 6 in., and the thickness of the material is 1 in., find the number of cubic inches of material.

36. The area of a chess board, having 8 squares along each side, is 80 square inches. Find the length of a side of one of its squares.

37. A rectangular field is 40 rods in length and 30 yards in width. Find in feet the side of a square field of equal area.

38. One side of a quadrilateral field measures 25 rods, the side opposite and parallel to it measures 36 rods, and the distance between the two sides is 12 rods. Find the area.

39. The sides of a triangle are 13, 14 and 15 feet, respectively. Find the perpendicular from the opposite angle, on the 15 foot side.

40. What is the superficial area of the outside of a closed box whose dimensions are 9, 10 and  $7\frac{1}{2}$  feet respectively?

41. Find the side of a square which is equivalent in area to a circle, whose circumference is 55 inches.

42. If it costs \$448 to fence a square field at \$4.40 a rod, what would it cost to fence the same amount of land in the form of a rectangle whose sides are in the ratio of 9 to 16?

43. A square and a rectangular field have the same perimeter, 100 yards. The length of the rectangular field is 4 times its width. Which contains the greater area, and how much?

44. Find the cost of the material for fencing a square field containing 10 acres with a wire fence, if the wire costs 3 cents a yard, and there are five wires in the fence, and if the posts cost 8 cents apiece, and are placed 8 feet apart.

45. A cubic foot of gold is extended by hammering so as to cover an area of 3 acres. Find, correct to 7 places of decimals, its thickness as a decimal of an inch.

46. The difference between the diameter and the circumference of a circle is 12 feet. Find its area.

47. Find the diagonal of a square whose area is 14 square inches.

48. A garden roller is 3 feet  $7\frac{1}{2}$  inches wide, and 5 feet  $10\frac{3}{4}$  inches in circumference. How much ground does it pass over in making three complete revolutions?

49. Find the number of cords of wood in a cylindrical stick of timber, the length being 40 feet and the circumference 22 feet.

50. Each edge of a cube is diminished by  $\frac{1}{8}$  of itself. By what fraction of itself is the volume diminished?

51. How many rows of desks can be arranged lengthwise in a room 24 feet wide, if each desk is 20 inches wide, and the spaces along the walls and between the rows are each to be, at least, 2 feet wide?

52. One extremity of a string is fastened to the corner of a square, edge 3 inches, and the string is then wound around the square; if the string is kept stretched in unwinding it, find the area of the space inclosed by the string after it is unwound and the line marked out by its extremity in one complete revolution.

53. A factory has 150 windows, 90 of which severally contain 16 panes, each pane being 8 inches by 12 inches, and the remaining windows severally contain 12 panes, each pane being 7 inches by 14 inches. Find the cost of glazing the whole at 25 cents per square foot.

54. How many planks, 2 inches thick, can be sawed from a log 10 feet in circumference, allowing  $\frac{1}{4}$  of an inch for each saw cut, and 2 slabs, each at least 5 inches thick, to be cast aside?

55. Find the length of an arc which subtends an angle of  $75^\circ$  at the centre of a circle of 12 inch radius.

56. Find the area in acres of a quadrilateral, whose diagonal is 80 chains, and the perpendicular from the opposite angles to it 29 chains and 23 chains respectively.

57. Find the area of the sector of a circle whose radius is 60 yards, the arc of the sector being 280 yards in length.

58. The area of a sector of a circle is 230 sq. ft.; the angle of the sector is  $50^\circ$ . Find the whole perimeter of the sector.

59. If a pipe, 3 inches in circumference, can fill a cistern in 2 hours, in what time can a pipe,  $4\frac{1}{2}$  inches in circumference, fill it?

60. The area of a sector is 115 sq. in., the area of the circle is 275 sq. in. Find the arc of the sector.

61. Find how many cubic feet of earth must be excavated to make a well 3 feet in diameter, and 30 feet deep.

62. The minute hand of a clock is 11 feet long. How many yards per day does its extremity travel?

63. A flat roof is 17 ft. 4 in. long, and 13 ft. 4 in. wide. Find the cost of covering it with sheet lead  $\frac{1}{8}$  of an inch thick, supposing that a cubic inch of lead weighs 6.5 ounces, and that 1 pound of it costs 7 cents.

64. A gas-jet is 11 feet above the pavement. How far must a man, who is 5 feet 10 inches high, stand from it so as to cast a shadow 7 feet long?

65. The distance to the top of a certain mountain is  $1\frac{1}{2}$  miles, and the circumference of the base 4.7 miles. What is the area of its surface, supposing it to be a cone?

66. The shadow of a man standing upright and 5 ft. 10 in. high, was measured and found to be 7 ft. 5 in.; the shadow of a pole, measured at the same time, was found to be 29 ft. 8 in. Determine the length of the pole.

67. Find the volume of a frustum of a cone, the radii of whose ends are 7 ft. and 12 ft. 4 in. respectively, and whose altitude is 9 feet.

68. The depth of water is 7 feet in a circular cistern, the circumference of whose base is 20 feet. Find the depth of the same quantity of water in another cistern, the perimeter of whose square base is 20 feet.

69. What is the volume of a frustum of a square pyramid, the sides of whose bases are 18 and 25 feet respectively, and the altitude 15 feet?

70. How many square yards of canvas will be required to make a conical tent 9 feet high and having a base of 4 feet radius, no allowance being made for seams?

71. A cone is 14 inches in height; if a cone 7 inches in height be cut from the top of it what fraction of the original cone will remain?

72. A circle and an equilateral triangle have equal perimeters, each being 36 inches; find the difference of their areas.

73. What is the volume of a sphere whose surface is 616 square inches?

74. The sides of a triangle are 13, 14 and 15 inches, respectively. If squares are described on the sides towards the outside of the triangle, find the perimeter of the figure and the whole space occupied by it.

X 75. Find the solid contents of a rubber ball, 4 inches in diameter, which has a hollow in it 2 inches in diameter.

X 76. Find the volume of the greatest sphere which can be cut out of a cubical block, whose edge is 3 inches.

X 77. A water-wheel, whose diameter is 14 feet, makes 50 revolutions per minute. Find, approximately, the number of miles per hour traversed by a point on the circumference of the wheel.

X 78. The driving wheel of a locomotive-engine, of diameter 7 feet, makes  $1\frac{1}{2}$  revolutions in 1 second. Find, approximately, the number of miles per hour at which the train is going.

X 79. How many pieces of money,  $\frac{3}{4}$  of an inch in diameter and  $\frac{1}{8}$  of an inch thick, can be coined from material in the form of a cube, whose edge is 3 inches?

80. Find the weight of gunpowder required to fill a hollow sphere 9 inches in diameter, supposing that 30 cubic inches of gunpowder weigh one pound.

81. The surface of a sphere is equal to that of a right circular cylinder; the radius of the base of the cylinder is 4 inches, and the height 1 foot. Find the volume of the sphere.

X 82. A solid is in the form of a right circular cylinder, with hemispherical ends; the extreme length is 42 inches and the diameter is 5 inches. Find the volume.

83. The height of a cone is 7 inches and its curved surface has three times the area of its base. Find the volume.

84. The radius of a circle is 12 feet; two parallel chords are drawn on opposite sides of the centre, one subtending at the centre an angle of  $60^\circ$ , and the other an angle of  $90^\circ$ . Find the area of the zone between the chords.

85. What is the length of a side of the greatest cube which can be cut from a sphere 1 inch in diameter?

86. Find the whole surface of a hemispherical bowl whose inner diameter is 4 inches, and outer 6 inches.

87. Compare the volumes of two spheres whose radii are in the ratio of 2 to 3.

88. A canal 12 miles long has an average width of 7 yards, and is 5 feet deep; how soon would the excavation of it be completed by 400 men, each removing, on the average, 20 cubic yards per day?

89. Find the cost of a leaden pipe of  $1\frac{1}{2}$  inch bore, which is  $\frac{1}{2}$  inch thick and 20 feet long, at 8 cents per pound, supposing that a cubic foot of lead weighs 11500 ounces.

90. A box with a lid is made of planking  $1\frac{1}{2}$  inches thick. If the external dimensions be 3 ft. 4 in., 2 ft. 6 in., and 1 ft. 8 in., respectively, how many square feet of planking, surface measure, does it contain?

91. The radius of a circle is 26 inches; the perpendicular, drawn from the centre on a chord, 10 inches. Find the length of the chord.

92. The radius of a circle is 4 feet; from a point 7 feet from the centre, a tangent is drawn to the circle. Find the length of the tangent.

93. The diagonals of a rhombus are 30 and 35 feet respectively. Find its area.

94. The parallel chords of a circular zone are 12 and 16, and its breadth 14. What is the diameter of the circle?

95. The side of a square is 40 yards, its corners are cut off so as to form a regular octagon (8 sides). What is the area of the octagon?

96. The radius of a circle is 8 feet; two parallel chords are drawn, each equal to the radius. Find the area of the zone between the chords.

A cubic foot of water weighs  $62\frac{1}{2}$  pounds; a gallon of water weighs 10 pounds.

97. Find the number of gallons of water in a tank which is in the form of a rectangular parallelepiped, 8 feet wide, 10 feet long, and 9 feet deep, if it is full of water.

98. How many gallons of water will a cistern contain, the diameter of whose base is 8 feet, and which is 4 feet deep?

99. Iron being nearly 8 times as heavy as an equal volume of water, find the weight of a solid sphere of iron whose radius is 3 inches.

100. A pond whose area is 4 acres, is frozen over with ice 6 inches in thickness. If water expands  $\frac{1}{10}$  in freezing, find the weight of the whole of the ice in tons.

101. The earth which is excavated from a cellar is twice as heavy as an equal volume of water. Find the weight of earth removed in digging a cellar 40 feet long, 32 feet wide, and 8 feet deep.

102. How many pails of water may be contained in a cylindrical cistern 4 feet deep, having a diameter of 6 feet, if each pail contains 12 quarts?

103. What is the depth of a 6 gallon pail that is 12 inches across, the sides being upright?

104. Find how many gallons are contained in a vessel, which is in the form of a right circular cone, the radius of the base being 8 feet, and the slant side 12 feet.

105. The radius of the base of a cylindrical vessel is 14 inches; a block of stone is placed in the vessel and is covered with water; on removing the block the level of the water sinks 4 inches. Find the weight of the block of stone, supposing that it is 8 times as heavy as an equal volume of water.

106. A cask full of water weighs 480 lb.; the cask when empty weighs 31 lb. Find the number of gallons the cask will hold.

107. How fast must the water rise in a well whose diameter is 7 feet, so that it may remain the same depth when a pump is emptying it at the rate of  $\frac{1}{4}$  of a ton of water per hour?

108. How many gallons of water will a circular vat contain that measures 12 feet across the bottom, 15 feet across the top, and 6 feet deep?

### PROBLEMS INVOLVING METRIC UNITS

109. The radius of a wheel is 1.4 m.; how many times will it revolve in going 55 Km.? ( $\pi = \frac{22}{7}$ .)

110. Find the number of square metres of carpet required to cover a floor, the dimensions of which are 6 m. 1.75 dm. by 4 m. 12 cm.

111. Find in metres the side of a square of which the area is 15227.56 sq. metres.

112. The area of a square field is 18.49 Ha.; find the perimeter in metres.

113. Find in centiares the whole surface of a cube, the volume of which is 27 cubic metres.

114. A parallelepiped whose edges are proportional to 2, 3 and 4 contains 3 cubic metres; find its whole surface in centiares.

115. A block of wood contains 4.5 cubic metres; find in ares the area it will cover if cut into sections 1 cm. thick.

116. Find in cubic metres the volume of a sheet of ice covering a pond of area 20 Ha. and thickness 3 cm.

117. How many ares are there in a square field whose diagonal is  $10\sqrt{2}$  metres?

118. The sides of a rectangular field are proportional to 2 and 3, its area is 24 Ha.; find the length of its diagonal in metres.

119. A rectangular garden 15m. long and 12m. broad is surrounded by a path 1.5m. wide; find the area of the path in centiares.

120. A circular area of 7 metres radius is divided into two portions in the ratio of 3 to 4; express the area of the smaller portion in ares.

121. Express in steres the volume of a cube whose edge is 3 metres.

122. The volume of a cube is 1 Ks.; find the length of its edge in centimetres.

123. How many steres of wood in a rectangular floor 40 m. long and 30 m. wide, the flooring being 2 cm. in thickness?

124. The excavation of a cellar, the dimensions of which are in the ratio 1, 4 and 5, cost \$54, at 10 cents per stere; find its dimensions.

125. One stere of water is poured into an empty tank 2 m. long and 1.25 m. wide; find the depth of the water in the tank.

126. An empty tank is 4 m. long and 2.5 m. wide; water is poured in at the rate of 1 stere per minute; in what time will the water be 1 dm. deep?

127. The dimensions of a rectangular tank are 3 m., 2 m., 1.5 m.; in what time would it be filled from two taps, the one pouring in 5 l. in 3 sec., and the other 5 Dl. in 20 sec.?

128. A cylindrical log has radius 1 m. 4 dm.; what length of it must be cut off to contain 61.6 steres?

129. A cubical box, made of material 5 cm. in thickness, is filled with 1 stere of water; find the amount of material in the box.

130. Find the depth of a cistern, the length and breadth being equal and double the depth, which will contain four million litres.

131. Find in centiares the area of a triangle, the sides of which are 1.3, 1.4, 1.5 metres respectively.

132. The parallel sides of a trapezium are 2.6 m., and 1.4 m.; find the perpendicular distance between these sides, given that the area is .75 centiares.

133. Find in litres the volume of a conical vessel, the diameter of the base being 21 cm., and the height 1 dm.

134. Find in kilogrammes the weight of a pyramid of lead, 6 dm. high, having a square base the side of which is 40 cm.; 1 cubic cm. of lead weighs 11.4 grammes.

## Miscellaneous Theorems and Applications

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It is frequently useful to know when a given number is exactly divisible by certain simple numbers, or what the remainder is when it is not divisible.

### Divisibility by 2, 4, 5 and 25.

Take any number, say, 42678.

$$42678 = 4267 \times 10 + 8.$$

Since 10 is divisible by 2, 5 or 10, it follows that  $4267 \times 10$  is also divisible by these numbers.

Therefore the remainder on dividing a number by 2, 5 or 10 is the same as on dividing its units' digit.

Therefore any number is divisible by 10 if its units' digit is 0 by 5, if its units' digit is 0 or 5; by 2, if its units' digit is 0 or any even digit.

If the number be written  $426 \times 100 + 78$ , it follows that the remainder on dividing it by 4, 20, 25, 50 or 100 is the same as the remainder obtained on dividing the number formed by the last two digits taken in order.

Thus the remainder on dividing 42678 by 4 is 2; by 20, 18; by 25, 3; by 50, 28; by 100, 78.

### Divisibility by 9 and 3.

$$\begin{aligned} 42678 &= 4 \times 10000 + 2 \times 1000 + 6 \times 100 + 7 \times 10 + 8 \\ &= 4(9999 + 1) + 2(999 + 1) + 6(99 + 1) + 7(9 + 1) + 8 \\ &= 4 \times 9999 + 2 \times 999 + 6 \times 99 + 7 \times 9 + (4 + 2 + 6 + 7 + 8). \end{aligned}$$

Since 9, 99, etc., are divisible by 9, it follows that the remainder on dividing the number by 9 is the same as the remainder on dividing  $(4+2+6+7+8)$  by 9, that is, on dividing the sum of the digits by 9.

Therefore any number is divisible by 9 if the sum of its digits is divisible by 9.

Also, if from any number the sum of its digits be subtracted, the remainder is divisible by 9.

This method of obtaining the remainder, without actually dividing by 9, is called “casting out the nines.”

Thus, when the nines are cast out of the number 42793584 the result is 6, which will be found to be the remainder on dividing the number by 9.

In finding the remainder the nines should be cast out as they occur in the process of adding.

This method of casting out the nines is frequently used for testing the correctness of the results in addition and multiplication.

Take any two numbers, say, 3245 and 4931.

The remainder on dividing the first by 9 is 5, and the second 8.

$$\begin{aligned}\therefore 3245 &= \text{a multiple of } 9, + 5 \\ \text{and } 4931 &= \text{a multiple of } 9, + 8 \\ \therefore \text{their sum} &= \text{a multiple of } 9, + (5+8).\end{aligned}$$

Therefore when the sum of the numbers is divided by 9 the remainder is the same as when the sum of the separate remainders is divided by 9.

The product of the same two numbers will be a multiple of  $9, + (5 \times 8)$ .

Therefore the product when divided by 9 will give the same remainder as when the product of the separate remainders is divided by 9.

By a similar method any number may be shown to be divisible by 3 when the sum of its digits is divisible by 3.

### LONGITUDE AND TIME

The **Longitude** of any place on the earth's surface is its distance, expressed in degrees, east or west from the **Prime Meridian**.

The prime meridian adopted by most nations is the one through the Royal Observatory at Greenwich, England.

When we say that the longitude of a place is  $25^{\circ}$  W., we mean that it is situated  $25^{\circ}$  west of the prime meridian.

The greatest longitude a place can have is  $180^{\circ}$  E. or  $180^{\circ}$  W.

Since the earth rotates on its axis once in every 24 hours, the place in which we live will pass through  $360^{\circ}$  in the same time.

That is, to 24 hours' difference in time will correspond  $360^{\circ}$  difference in longitude.

Therefore 1 hour's difference in time corresponds to  $15^{\circ}$  difference in longitude.

Since the earth rotates from west to east, when it is noon at any place, it is an hour earlier, or 11 a.m. at a place  $15^{\circ}$  west, or an hour later, or 1 p.m., at a place  $15^{\circ}$  east.

If the difference in longitude of two places is  $18^{\circ} 15'$ , what is the difference in time?

Since  $15^\circ$  difference in longitude corresponds to 1 hour's difference in time,

$\therefore 1^\circ$  difference in longitude corresponds to 4 min. difference in time,

$\therefore 18\frac{1}{4}^\circ$  difference in longitude corresponds to  $(4 \times 18\frac{1}{4})$  min. difference in time,

$\therefore$  the difference in time is 73 min. or 1 hr. 13 min.

The longitude of Montreal is  $73^\circ 35'$  W.; what is the difference in time of Montreal and Greenwich?

As before the difference in time will be  $(4 \times 73\frac{7}{8})$  min. = 4 hr. 54 min. 20 sec.

**Standard Time.**—When the difference in longitude of any two places is a multiple of  $15^\circ$ , the difference in time is an integral number of hours.

To obviate the necessity of considering such longitudes as that of Montreal, in the preceding problem, a system of standard time has been adopted by nearly all civilized nations.

By this system, for the purpose of time, all places in one section are considered as having the same longitude some multiple of  $15^\circ$ , so that the difference in time of any two places is always an integral number of hours.

The standard time of any place is the same as the time indicated by the nearest of the following meridian lines,  $0^\circ$ ,  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ , etc.

Thus, for the purpose of time the longitude of all Great Britain is  $0^\circ$ , of Germany and Italy  $15^\circ$  E., of South Africa  $30^\circ$  E., of Newfoundland and Eastern Canada  $60^\circ$  W., of Western Canada  $120^\circ$  W.

Thus, when it is noon in England it is 5 hours earlier in Montreal, 1 hour later in Rome, Italy; 6 hours earlier in Winnipeg, 8 hours earlier in Victoria, B.C., and 9 hours earlier in Dawson, Yukon.

When it is 12 o'clock noon at St. John, N.B., Atlantic time, it is 11 o'clock at Montreal, Eastern time; 10 o'clock at Winnipeg, Central time; 9 o'clock at Regina, Mountain time; and 8 o'clock at Vancouver, Pacific time.

The longitude of San Francisco is  $122^{\circ} 25' W.$ , of Hamilton  $79^{\circ} 54' W.$  When it is 2 p.m. in San Francisco what is the time in Hamilton?

For the purpose of time the longitude of San Francisco is  $120^{\circ} W.$ , and of Hamilton  $75^{\circ} W.$

$\therefore$  the difference in longitude is  $45^{\circ}$ ,

$\therefore$  the difference in time is 3 hours,

$\therefore$  when it is 2 p.m. at San Francisco it is 3 hours later, or 5 p.m. in Hamilton.

The longitude of Toronto is  $79^{\circ} 24' W.$  What is the difference between the true and the standard time?

The difference between the true and the standard time will be the difference in time corresponding to a difference of  $4^{\circ} 24'$  in longitude, and is therefore 17 min. 36 sec. The standard time of Toronto is therefore 17 min. 36 sec. faster than the true time.

## RATIO AND PROPORTION

**Ratio.**—The ratio of 3 to 4 is expressed in the form  $3 : 4$  or in the form  $\frac{3}{4}$ , either form meaning that  $3 = \frac{3}{4}$  of 4.

A fraction is merely a convenient way of writing a ratio, the terms of the ratio forming the numerator and denominator of the fraction.

Thus, the ratio of 8 in. to 12 in.  $= \frac{8 \text{ in.}}{12 \text{ in.}} = \frac{2}{3}$ .

The ratio of 4 days to  $4\frac{1}{2}$  hours  $= \frac{96}{4\frac{1}{2}} = \frac{64}{3}$ .

**Proportion.**—When two ratios are equal the four terms are said to form a proportion.

Thus,  $2 : 3 = 8 : 12$  because  $\frac{2}{3} = \frac{8}{12}$ .

It is sometimes written in the form  $2 : 3 :: 8 : 12$  and is read 2 is to 3 as 8 is to 12.

Here 2 and 12 are called the extremes and 3 and 8 the means.

Since  $\frac{2}{3} = \frac{8}{12} \therefore \frac{2 \times 12}{3 \times 12} = \frac{8 \times 3}{12 \times 3}$

Since the denominators are equal the numerators must be equal.

Therefore in a proportion, the product of the extremes is equal to the product of the means.

Divide \$300 between A and B in the ratio of 5 to 7.

For every \$5 that A receives B will receive \$7,

$\therefore$  out of \$12 divided A will receive \$5,

$\therefore$  A receives  $\frac{5}{12}$  of \$300 = \$125,

and B receives  $\frac{7}{12}$  of \$300 = \$175.

Similarly, when a number is divided into 3 parts in the proportion of 4 : 5 : 8, the first part is  $\frac{4}{17}$  of the number, the second  $\frac{5}{17}$  and the third  $\frac{8}{17}$ .

Find a number which will be in the same ratio to 25 that 3 is to 7.

The required number :  $25 = 3 : 7$ ,

$\therefore$  7 times the required number  $= 25 \times 3$ ,

$\therefore$  the number is  $\frac{25 \times 3}{7} = 10\frac{5}{7}$ ,

that is,  $10\frac{5}{7} : 25 = 3 : 7$ .

**Mixtures.**—If teas worth 45c. and 55c. per lb. are mixed in the ratio of 4 : 5, find the value per lb. of the mixture.

If 4 lb. of the first kind are used the value is \$1.80.

Then 5 lb. of the second must be used and the value is \$2.75.

$\therefore$  9 lb. of the mixture are worth \$4.55,

$\therefore$  the value per lb. of the mixture is  $\frac{\$4.55}{9} = 50\frac{5}{9}\text{c.}$

In what ratio must teas worth 28c. and 42c. per lb. be mixed so that the mixture may be worth 37c. per lb.?

As each lb. of 28c. tea is increased in value 9c.,

and each lb. of 42c. tea is decreased in value 5c.,

$\therefore$  in order that the increases and decreases may equalize each other, with every 5 lb. of the first kind we must mix 9 lb. of the second.

$\therefore$  they must be mixed in the ratio of 5 : 9.

### MOTION IN THE SAME OR IN OPPOSITE DIRECTIONS

**Relative Velocity.**—If two persons travel from the same point at rates of 5 and 3 miles per hour respectively, they will separate at the rate of 2 miles per hour if they travel in the same direction. Their relative velocity is said to be 2 miles per hour. If they travel in opposite directions their relative velocity will be 8 miles per hour.

If A is 100 yards ahead of B and if A travels 5 yards while B travels 7 yards, how far will B travel before he overtakes A?

Here B must gain 100 yards on A.

Since B gains 2 yards when he travels 7 yards,

$\therefore$  B gains 100 yards when he travels 350 yards,

$\therefore$  when B has gone 350 yards he has overtaken A.

The lengths of two trains are 90 yards and 130 yards, and their rates are 25 and 35 miles per hour respectively. In what time will they pass when they are moving (1) in opposite directions, (2) in the same direction?

(1) Here, as in all problems, where the relative motion of two bodies is considered, attention should be fixed on some particular point in each body.

When the trains move in opposite directions, the most convenient points to consider would be the front of each engine.

When these two points are opposite each other, the trains will begin to pass and they will have completely passed when the ends of the trains are opposite; that is, when the two trains have together travelled a distance of  $90 + 130$  or 220 yards.

They together travel 60 miles in 1 hour,

$\therefore$  they travel 220 yards in  $\frac{220 \times 60 \times 60}{60 \times 1760}$  sec. or  $7\frac{1}{2}$  sec.

(2) When they move in the same direction, they begin to pass when the front of the faster train is opposite the rear of the slower.

To completely pass, the faster train must gain 220 yards on the slower.

The faster train gains 10 miles in 1 hour,

$\therefore$  it gains 220 yards in  $\frac{220 \times 60 \times 60}{10 \times 1760}$  sec. or 45 sec.

The rate of a stream is 2 miles per hour. If a man can row down stream at the rate of 7 miles per hour, at what rate can he row up stream?

Since the stream carries him down 2 miles each hour his rate in still water will be 5 miles per hour.

In rowing up stream, the stream retards him 2 miles each hour,

$\therefore$  he can row up stream at 3 miles per hour.

The difference between the rates of rowing up and down stream is therefore **twice** the rate of the stream.

If I can row 10 miles up stream in 5 hours and back again in 2 hours, what is the rate of the stream?

The rate of rowing up stream is 2 miles per hour.

The rate of rowing down stream is 5 miles per hour.

Since the difference is 3 miles per hour, the rate of the stream is  $1\frac{1}{2}$  miles per hour.

**Circular Motion.**—If A and B run around a circular track at rates which are in the ratio of 5 to 3, where will they be first together if they start at the same place and run in the same direction?

They will be first together when A has gained on B one complete round.

When A goes 5 yards B will go 3 yards,

$\therefore$  A gains 2 yards when he has gone 5 yards,

$\therefore$  A gains 2 rounds when he has gone 5 rounds,

$\therefore$  A gains 1 round when he has gone  $2\frac{1}{2}$  rounds.

They will therefore be first together when A has gone  $2\frac{1}{2}$  rounds, that is, they will be first together half way round the track.

The hands of a clock are together at 12 o'clock. When are they next together?

The minute hand goes 60 minute spaces in 1 hour; the hour hand goes 5 minute spaces in 1 hour,

$\therefore$  in 60 min. the min. hand gains 55 min. spaces on the hour hand.

In order that the hands may be together the min. hand must gain 60 min. spaces on the hour hand, which it will do in  $\frac{60}{55}$  of 60 min. or  $65\frac{5}{11}$  min.

$\therefore$  they are next together at  $5\frac{5}{11}$  min. past one o'clock.

At what time between 4 and 5 are the hands of a clock at right angles?

At 4 o'clock there are 20 min. spaces between the hands. They will be at right angles when they are 15 min. spaces apart.

$\therefore$  the min. hand must gain 5 min. spaces on the hour hand.

Since it gains 55 spaces in 60 min.,

it will gain 5 spaces in  $\frac{5}{55}$  of 60 min. or  $5\frac{5}{11}$  min.

$\therefore$  the hands are at right angles at  $5\frac{5}{11}$  min. past 4.

They will also be at right angles when the min. hand is 15 spaces ahead of the hour hand.

To be 15 min. spaces ahead of the hour hand, the min. hand must gain  $(20+15)$  min. spaces on the hour hand, which it will do in  $\frac{35}{55}$  of 60 min. or  $38\frac{2}{11}$  min.

$\therefore$  the hands are again at right angles at  $38\frac{2}{11}$  min. past 4.

## WORK AND TIME

If A can do a piece of work in 10 days and B in 15 days, in what time can they do it if they work together?

A can do  $\frac{1}{10}$  of the work in 1 day,  
 or a day's work for A is  $\frac{1}{10}$  of the work,  
 and a day's work for B is  $\frac{1}{15}$  of the work,  
 $\therefore$  a day's work for A and B is  $(\frac{1}{10} + \frac{1}{15})$  or  $\frac{1}{6}$  of the work.  
 $\therefore$  A and B together can do the work in 6 days.

A cistern has three pipes. The first fills it alone in 12 hours, the second in 15 hours, and the third empties it in 10 hours. If the cistern is empty and all the pipes are opened together, in what time will it be filled?

The first pipe fills  $\frac{1}{12}$  of the cistern in 1 hour.

The second pipe fills  $\frac{1}{15}$  of the cistern in 1 hour.

The third pipe empties  $\frac{1}{10}$  of the cistern in 1 hour.

$\therefore$  the three pipes will fill  $(\frac{1}{12} + \frac{1}{15} - \frac{1}{10})$  or  $\frac{1}{20}$  of the cistern in 1 hour.

$\therefore$  the cistern will be filled in 20 hours.

A and B are paid \$60 for certain work. If their rates of working are in the ratio of 3 : 4, and the number of days they work in the ratio of 7 : 6, how should the money be divided?

Suppose A is paid \$3 per day, then B should receive \$4 per day.

Suppose A works 7 days and therefore earns \$21, then B must work 6 days and therefore earns \$24.

$\therefore$  the amount received for the whole work should be divided between them in the ratio of 21 : 24, or 7 : 8.

$\therefore$  A should receive  $\frac{7}{15}$  of \$60 or \$28,  
 and B should receive  $\frac{8}{15}$  of \$60 or \$32.

### SPECIAL UNITS OF MEASUREMENT

**Lumber.**—The unit used in measuring lumber is the board foot.

A board foot, or a foot of lumber, is 1 foot long, 1 foot wide and 1 inch thick. It is therefore  $\frac{1}{12}$  of a cubic foot.

The price of lumber is usually quoted as so much per thousand (M) feet, board measure.

A board 10 ft. long, 1 ft. wide and 1 in. thick contains 10 ft. of lumber, for it is evidently 10 times the unit, 1 board foot.

A plank is 18 ft. long, 8 in. wide and 3 in. thick. How much lumber does it contain, and what is its value at \$75 per 1000 feet?

The length is here 18 times the length of the unit.

The width is  $\frac{2}{3}$  of the width of the unit.

And the thickness is 3 times the thickness of the unit.

$\therefore$  the number of ft. of lumber is

$$18 \times \frac{2}{3} \times 3 \text{ or } 36.$$

If 1000 feet cost \$75,

36 feet will cost  $1\frac{36}{1000}$  of \$75 or \$2.70.

**Carpet.**—The unit of measurement is a linear yard. Carpet is usually 27 in. or  $\frac{3}{4}$  of a yard in width, so that a yard of carpet is usually only  $\frac{3}{4}$  of a square yard.

If a room is 27 feet long and 18 feet wide, how much carpet 27 in. wide will be required to cover the floor?

If the carpet is laid lengthwise the number of strips required will be  $18 \text{ ft.} \div \frac{3}{4} \text{ yd.}$  or 8.

Since it requires 8 strips, each 27 feet or 9 yards in length, the number of yards of carpet is 72.

If the carpet is laid crosswise the number of yards required would, in this case, be the same.

If a room is 22 feet long and 19 feet wide, how much carpet is required?

If laid lengthwise, the number of strips required is  $19 \text{ feet} \div \frac{3}{4} \text{ yd.}$  or  $8\frac{4}{9}$ .

Since fractional widths cannot be bought, this must be considered as 9 strips.

It will therefore require 9 strips each 22 feet long, or 66 yards of carpet.

If the carpet, in this case, is laid crosswise, 10 strips would be required, each 19 feet long, and therefore the number of yards would be  $\frac{10 \times 19}{3}$  or  $63\frac{1}{3}$  yards.

Since carpet is usually made with a pattern, and the figures of the pattern should be matched in adjacent strips, it follows that there may be some waste to be allowed for in each strip.

Thus, in the previous problem if the waste in matching be 6 in. in each strip, then this amount must be added to the length of each strip.

**Wall Paper.**—The price of wall paper is usually given as so much per roll. A single roll contains 8 linear yards and a double roll 16 linear yards, and the paper is generally 18 in. wide.

In finding the number of rolls of paper required for the walls or ceiling of a room the number of strips should first be obtained in the same manner as the number of strips of carpet is found. In finding the number of strips

required for the walls, a deduction should be made for the width of doors and windows.

In the result, any fractional part of a roll should be considered as a whole roll.

### EXERCISE

1. Show that a number is divisible by 6 when its last digit is divisible by 2, or is zero, and the sum of its digits is divisible by 3.

2. Show that a number is divisible by 8 when its last three digits taken in order form a number divisible by 8, or are zeros.

3. What is the test for the divisibility of a number by 15? by 18? by 125?

4. A number, formed by writing down three digits and then repeating them in order, is divisible by 1001 and therefore by 7, 11, and 13. Thus,  $247247 = 247 \times 1001$ .

5. Show that the difference between two numbers consisting of the same digits, arranged in different order, is divisible by 9.

6. A number formed by writing any units' digit and placing before it double the units' digit is always divisible by 3 and 7.

7. What is the remainder on dividing 1234567 by 2, 3, 4, 5, 8, 9, 10, or 25?

8. Find the product of 42752 and 327, and verify the work by casting out the nines.

9. Find the remainder on dividing the product of 123, 456, and 789 by 9.

10. Write down any two numbers made up of the same digits and find their difference. If one digit in the remainder be struck out, how may I know by examining the rest of the remainder what digit was struck out? When does this test fail?

11. When it is noon in London, Eng., what is the longitude of a place in which it is 3 p.m.? 5 a.m.?

12. A ship's chronometer keeps Greenwich time. When it is noon, local time, the chronometer indicates 3.32 p.m. What is the longitude of the ship?

13. What is the difference in true time between Halifax,  $63^{\circ} 36' \text{ W.}$ , and Vancouver,  $123^{\circ} 5' \text{ W.}$ ?

14. The difference in true time between Winnipeg  $97^{\circ} 7' \text{ W.}$  and Toronto, is 1 hr. 10 min. 52 sec. What is the longitude of Toronto?

15. At 10.24 a.m. a steamer in longitude  $25^{\circ} 16' \text{ W}$  sends a wireless message to another steamer. It is received at 10.19 a.m., true time in each case. What is the longitude of the second steamer?

16. Find the longitude of the Falkland Islands, if it is 5 o'clock a.m. there, when it is 1 o'clock p.m. at Ras el Had, the longitude of which is  $60^{\circ}$  east.

17. What is the difference in standard time between Hamilton, Ont.,  $79^{\circ} 54' \text{ W.}$ , and Dawson, Yukon,  $139^{\circ} 20' \text{ W.}$ ?

18. The San Francisco earthquake occurred on April 18th, 1906, at 5 a.m. If 20 min. be allowed for transmission, when did the news reach London, Eng.? (The time meridian of San Francisco is  $120^{\circ} \text{ W.}$ )

19. What is the difference between the true and the standard time of Kingston, Ont., the longitude being  $76^{\circ} 29' \text{ W.}$ ? Of Pictou, N.S.,  $62^{\circ} 41' \text{ W.}$ ? Of Regina, Sask.,  $104^{\circ} 37' \text{ W.}$ ? In each case is the standard time faster or slower than true time?

20. What is the ratio of the time that 8 men will take to do a piece of work, to the time that 6 men will take to do the same work?

21. Divide \$1 into two parts in the ratio of 9 : 16.

22. What number is the same ratio to 154 that 45 is to 126?

23. Two men rent a pasture for \$68.75. One puts in 23 cattle and the other 32. How much should each pay?

24. Divide 1950 into 3 parts proportional to 5 : 11 : 23.

25. Two persons travelling together agree to pay expenses in the ratio of 8 to 5. The first (who contributes the greater sum) pays away on the whole \$79.65, the second \$24.35. What must one pay the other to settle the bill according to agreement?

26. A man has 50, 25, 10, and 5 cent pieces, the number of each being in the proportion of 1, 2, 3, and 4. If their total value is \$10.50, how many has he of each?

27. Divide \$3648 among three persons, so that the share of the first to that of the second shall be as 7 to 9, and of the first to the third as 3 to 4.

28. The sum of the ages of A and B is now 75 years, and their ages 18 years ago were as 2 to 1. Find their present ages.

29. Divide \$230 among 11 men and 13 boys so that a boy's share shall be to a man's share as 1 : 3.

30. One cask contains wine and water in the ratio of 6 : 1, another in the ratio of 5 : 1. If equal quantities be taken from each, and mixed together, what will be the ratio of wine to water in the mixture?

31. A grocer mixes three kinds of tea worth 25c., 30c. and 45c. a lb. in the proportion of 3 : 2 : 1 and sells the mixture at 40c. a lb. What does he gain on every 100 lb. he sells?

32. In what ratio must coffee worth 31c. a lb., be mixed with chicory worth 13c. a lb., to make a mixture worth 28c. a lb.?

33. How much water is there in a mixture of 100 gallons of wine and water, worth \$1.00 a gallon, if the wine cost \$1.20 a gallon?

34. How much water will dilute 9 gal. 1 qt. 1 pt. of alcohol  $\frac{96}{100}$  strong to  $\frac{84}{100}$ ?

35. Divide \$338 between A and B, such that 3 times A's share may be \$127 less than 4 times B's.

36. A man paid \$162 to 54 labourers, consisting of men and boys; each man received \$5 and each boy 50 cents. How many were there of each?

37. In a 400 yard race A wins, B being 10 yards, and C 40 yards behind. How much would B beat C in a mile race?

38. A and B run a hundred yard race; A takes 7 steps while B takes 9, but 11 of A's steps are equal in length to 13 of B's. Which will win the race, and by how much?

39. Two men start together to travel in the same direction around a circular track; their rates are as 11 to 14. Find where they will first be together again.

40. Three men start together to walk around a circular course 60 yards in circumference, their rates being 2,  $2\frac{1}{3}$ , and  $2\frac{1}{2}$  yards per second, respectively. Find when and where they will first be together again.

41. Two trains start at the same time from A and B, and proceed toward each other at the rates of 30 and 50 miles per hour, respectively. When they meet it is found that one train has travelled 135 miles farther than the other. Find the distance between A and B.

42. How long will a train 110 yards in length, and running at 30 miles per hour, take to pass a man walking in the same direction at 3 miles per hour? How long if the man is walking in the opposite direction?

43. A train 88 yards long overtook a person walking at the rate of 4 miles an hour and passed him completely in 10 seconds; it afterwards overtook another person and passed him in 9 seconds. At what rate per hour was this second person walking?

44. A man can row 5 miles an hour in still water. How far may he go down a stream which flows at the rate of 3 miles an hour, so that he may just take 2 hr. 40 min. for the round trip?

45. When are the hour-hand and the minute-hand of a clock first together after 2 o'clock?

46. When are the hands of a clock opposite to each other between 7 and 8 o'clock?

47. At what time, between 3 and 4 o'clock, is the minute-hand 5 minute spaces in advance of the hour-hand?

48. A clock which was 15 minutes fast at 11 a.m. on Wednesday, is exactly right at 4.30 p.m. on Friday. How many minutes will it be slow at 10 minutes to 7 p.m. on Saturday?

49. A can do one-half as much work as B; B can do one-half as much as C; together they can complete a piece of work in 12 days. In what time could each alone complete it?

50. A number of men engage to do a piece of work in 9 days. How long may 5 men remain away, and the work be finished in the same time by their bringing 10 men more with them?

51. A vessel can be emptied by 3 taps; by the first alone in 80 minutes, by the second alone in 200 minutes, and by the third alone in 5 hours. In what time will the vessel be emptied if the three taps are opened together?

52. If 15 men, 18 women and 26 boys receive \$64.40 for a day's work, and 2 men receive as much as 3 women or 10 boys, what does each man, woman, and boy, respectively, receive for a day's work?

53. If 4 men or 7 boys can do a piece of work in 42 days, in what time can 7 men and 4 boys do it?

54. Two men working together can do a piece of work in 20 days. If the work is worth \$120, and one man works 7 days less than the other, how should the money be divided?

55. Two men and 5 boys can do a piece of work in 20 days; 1 man and 8 boys can do it in 18 days. In what time can a man or a boy do it?

56. A can do as much work in 10 hours as B can do in 11, but he does not work the same time. If A earns \$29.70 and B earns \$30.00, compare the times they each worked.

57. A and B work  $5\frac{1}{2}$  and  $7\frac{1}{2}$  hours a day, respectively, for one day and receive the same wages; on the second day they work  $6\frac{1}{4}$  and  $8\frac{1}{4}$  hours a day, respectively; they receive \$13.98 for their two days' work. How should it be divided?

58. A can do a piece of work in 10 days, B in 12, C in 15. They all begin together, but only C continues till the work is finished, A leaving 3 days, and B  $2\frac{1}{4}$  days before completion. In what time is the work done?

59. How many board feet are there in a stick of timber 20 ft. long and 18 in. square?

60. Find the cost of 40 scantling 2 in. by 4 in. and 18 ft. long at \$18 per M (1000).

61. How much inch lumber is required for a floor 28 ft. by 32 ft. and what will it cost at \$25 per M?

62. A bridge 80 ft. long and 20 ft. wide, is covered with oak planks  $2\frac{1}{2}$  in. thick. What will it cost at \$32 per M?

63. How many feet of inch siding are needed for a house 28 ft. by 32 ft. and 18 ft. high, the siding being 5 in. wide and laid 4 in. to the weather, no allowance being made for gables or for doors and windows?

64. What will it cost at \$18 per thousand to fence a field 30 rods by 40 rods with one round of 9 in. boards and 5 of 6 in. boards, the boards being 1 in. thick?

65. A sidewalk is 440 yards long and 8 feet wide, and is made of plank 2 in. thick. The planks rest on three continuous lines of scantling 3 in. by 4 in. Find the cost of the lumber at \$15 per M.

66. How many yards of carpet 27 in. wide will be required for a room 21 ft. 9 in. long and 18 ft. 9 in. wide, the carpet running lengthwise and there being no waste in matching?

67. Find the cost of carpeting a room 14 ft. 8 in. by 18 ft. with 27 in. carpet, running lengthwise, and costing \$1.20 a yard, allowing 4 in. on each strip for waste in matching.

68. How many yards of 27 in. carpet are required for a room 17 ft. 6 in. by 15 ft. 5 in., if the strips run crosswise and 7 in. be wasted in matching each strip except the first one?

69. A room 20 ft. by 17 ft. is carpeted with a carpet 1 yd. wide (without a pattern), running lengthwise and costing \$1.20 a yard. It is surrounded by a carpet border 1 foot wide costing 75c. a yard. Find the total cost.

70. Find the cost of stair carpet at \$1.05 a yard for a flight of stairs of 20 steps, 11 in. wide with 7 in. rise.

71. How many strips of 18 in. paper are required for a room 30 ft. by 24 ft., if there are 4 windows and 2 doors, allowing 3 feet for each?

72. The walls of a room are 8 ft. 6 in. above the base-board, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, what will the paper cost at 40c. a single roll, the paper being 18 in. wide?

73. How many square yards of plastering are there in the walls and ceiling of a room 16 ft. by 20 ft. and 9 ft. high, deducting 14 sq. yd. for openings and base-board?

74. Find the cost of plastering a room 30 ft. by 27 ft. and 12 ft. high at 20c. a sq. yd., if there are two doors each 3 ft. by 7 ft. and three windows each 3 ft. 4 in. by 6 ft., the plasterer being allowed pay for one half the area of the openings for his trouble in plastering around them.

# Commercial Arithmetic

## PERCENTAGE

The term **Per Cent.** means **Hundredths.**

The fraction  $\frac{5}{100}$  or .05 is also written 5 per cent., or 5%.

A per cent. of any quantity is simply a fraction of that quantity, the denominator of the fraction being 100. Thus:

$$25\% \text{ of a quantity} = \frac{25}{100} \text{ or } .25 \text{ or } \frac{1}{4} \text{ of it.}$$

$$37\frac{1}{2}\% \quad " \quad " \quad = \frac{37\frac{1}{2}}{100} \text{ or } .375 \text{ or } \frac{3}{8} \text{ of it.}$$

$$62\frac{1}{2}\% \quad " \quad " \quad = \frac{62\frac{1}{2}}{100} \text{ or } .625 \text{ or } \frac{5}{8} \text{ of it.}$$

$$\frac{3}{4} \quad " \quad " \quad = \frac{\frac{3}{4} \text{ of } 100}{100} \text{ or } 75\% \text{ of it.}$$

$$\frac{2}{5} \quad " \quad " \quad = \frac{\frac{2}{5} \text{ of } 100}{100} \text{ or } 40\% \text{ of it.}$$

$$\frac{1}{7} \quad " \quad " \quad = \frac{\frac{1}{7} \text{ of } 100}{100} \text{ or } 14\frac{2}{7}\% \text{ of it.}$$

$$12\% \text{ of } \$30 = \frac{12}{100} \text{ of } \$30 = \$3.60.$$

45 is  $\frac{3}{4}$  or .75 of 60, therefore 45 is 75% of 60.

## EXERCISE

1. Express the following as fractions in their lowest terms: 20%, 35%, 40%,  $16\frac{2}{3}\%$ , 75%,  $87\frac{1}{2}\%$ ,  $85\frac{5}{7}\%$ ,  $37\frac{1}{2}\%$ , 100%,  $112\frac{1}{2}\%$ .

2. What per cent. is each of the following:  $\frac{3}{5}$ ,  $\frac{1}{6}$ ,  $\frac{4}{15}$ ,  $\frac{9}{40}$ ,  $\frac{2}{11}$ ,  $\frac{15}{8}$ , .25, .365, .0875?

3. What per cent. of 625 is 25? What per cent. of 12 is 9? What per cent. of 375 is 15?

4. What % of 100 is 4? What % of 200 is 6? What % of 250 is 5?

5. 7 is what % of 35? 12 is what % of 72? 95 is what % of 1900?  $12\frac{1}{2}$  is what % of 225?  $\frac{5}{6}$  is what % of  $17\frac{1}{2}$ ?

6. Find  $12\frac{1}{2}\%$  of 1728 men; of 864 bushels.

7. Find  $3\frac{5}{16}\%$  of \$144.16.

8. Find  $1\frac{1}{10}\%$  of \$225,000,000.

9. Find  $33\frac{1}{3}\%$  of 1260 marks; of 172.80 francs.

10. \$365 is what % of \$5840?

11. 36 minutes is what % of 1 day?

12. £3 2s. 6d. is what % of £25?

13. Ten per cent. of a certain number is 13; find the number.

14. The number of boys in a school is 60% of the number of girls. The number of girls is 60; how many pupils are there in the school?

15. Distilled water is composed of two gases,  $11\frac{1}{9}\%$  by weight being hydrogen and the remainder oxygen. Find the weight of each gas that can be obtained from 10 lb. of water.

16. The average attendance at a school this term is 225, which is an increase of  $12\frac{1}{2}\%$  on the average attendance of last term; what was the average attendance last term?

17. What number increased by 14% of itself is equivalent to 285?

18. What number diminished by  $11\frac{1}{4}\%$  of itself is equivalent to 710?

19. A crystal of copper sulphate weighing 5 grams is heated sufficiently to expel all the water from it and the residue weighs 3.196 grams. What per cent. of water did the crystal contain?

20. On account of the increase of value of flour the price of bread is advanced 25%. Formerly ten loaves were sold for one dollar; what number of loaves may now be had for fifty cents?

21. A regiment lost 20% of its men in a battle; 10% of the remainder deserted, there then remained 360 men. How many men were there originally in the regiment?

22. The number of Canadians in Detroit in 1900 was 25400, which was 26.3% of the foreign-born population. Find the number of the foreign-born population of Detroit in that year.

23. The earnings of a mill for two years amounted to \$6560; the earnings the 2nd year were 5% more than the earnings of the 1st year. Find the amount of the 2nd year's earnings.

24. Water expands 10% in freezing; how many cubic feet of ice will weigh 1 ton, given that 1 cu. ft. of water weighs  $62\frac{1}{2}$  lb.?

25. A bankrupt pays 30% of his debts; the amount that a creditor receives is what per cent. of his loss?

26. A house and lot, bought for \$4000, increased in value 120%; what was the increased value?

27. A teacher spends  $92\frac{1}{2}$ % of his salary, and has \$120 left each year; in how many years will he save one year's salary?

28. One number is double another; 10% of the greater and  $12\frac{1}{2}$ % of the less together make 39; what are the numbers?

29. The analysis of a specimen of lignite is given as follows: moisture, 29.07%; combustible matter, 28.96%; fixed carbon, 24.47%; ash, 17.50%. Find the weight of each in a ton of lignite.

30. South Africa imported 149,459,655 lb. of flour in 1907, of which Canada supplied 38,720,000 lb. What per cent. of the total imports of flour was from Canada?

31. A's money is  $33\frac{1}{3}$ % more than B's; how much per cent. is B's of A's?

32. A boy changed  $33\frac{1}{3}$ % of his paper money into silver; he spent 50% of this silver in buying a ball and bat, which cost \$1.50; how much money had he?

33. The number of girls in a school exceeds the number of boys by 50; the number of boys is  $37\frac{1}{2}$ % of the whole; find the number of girls.

34. A can do 10% of a piece of work in 1 day, and B  $12\frac{1}{2}$ %; what per cent. of the work will remain to be done after A has worked 2 days, and B 3 days?

35. A man who owned 30% of a mine sold  $33\frac{1}{3}\%$  of his share for \$12000; what is the value of the mine?

36. A farm cost \$4000; 60% of this sum was 40% of 3 times the value of the house that was built on the farm; what was the cost of the house?

37. Divide 1440 into three parts, so that 10% of the first part,  $12\frac{1}{2}\%$  of the second part, and  $16\frac{2}{3}\%$  of the third part may be equal.

38. One-fifth is what per cent. of one-half?

39. A owns 80% of a farm and B owns the remainder; C sells the farm for them and receives 4% of the selling price for his services; what per cent. of the selling price does A receive?

40. From a cask containing 126 gallons of wine, 2 gals.  $1\frac{3}{5}$  qt. leaked away; what per cent. was lost?

41. A druggist buys goods at \$5 per pound Av., and sells at \$5 per pound Troy; his gain is what per cent. of the cost?

42. Mr. Brown sold his farm for \$15840, which was 10% less than he gave for it, and he gave 10% more than it was worth; what was the actual value of the farm?

43. Two per cent. of a certain number, and 3% of half of the number, together make 21; what is the number?

44. Five per cent. of a certain number and 6% of twice that number, together make 175; find the former number.

## TRADE DISCOUNT

It is the custom of manufacturers and wholesale dealers to issue catalogues, with list prices at which their products are offered for sale.

From these prices one or more discounts are usually given.

Thus, if the catalogue or list price of a book is \$3.00 with a discount of 25%, the book would sell for \$3.00 less 25% of \$3.00, or \$2.25.

Here \$3.00 is the **List or Marked Price**, and \$2.25 is the **Selling Price**.

When dealers wish to change the selling prices of their goods, by reason of a difference in the cost of production, or in the expense of putting them upon the market, or for any other cause, they usually alter the rates of discount, the list prices remaining unchanged.

If they find it necessary to decrease the selling price, they may offer additional discounts or increase those already in use. If they find it necessary to increase the selling price, they may offer fewer discounts or decrease those already in use.

This method is more convenient and less expensive than that of issuing new catalogues and list prices.

If two or more discounts are allowed, the first discount is a certain per cent. of the list price; the second discount is a certain per cent. of the remainder after deducting the first discount; and so on if further discounts are given.

If a watch is listed at \$50, with discounts of 40%, 20% and 10%,

the first discount is 40% of \$50 or \$20,

∴ the first reduced price is \$30.

The second discount is 20% of \$30 or \$6,

∴ the second reduced price is \$24.

The third discount is 10% of \$24 or \$2.40,

∴ the selling price is \$21.60.

In this case the total discount is \$50 - \$21.60 or \$28.40.

If a single discount of 70% (40 + 20 + 10) were allowed, the discount would be \$35.

Why is a single discount of 70% not equal to the three successive discounts of 40%, 20% and 10%?

### EXERCISE

1. Find the selling price in each case, if the list prices and rates of discount are as follows:

(a) \$540; 20%. (b) \$1000; 25% and 10%.

(c) \$2500; 10%, 8% and 5%.

(d) \$1440; 25%,  $12\frac{1}{2}\%$  and  $16\frac{2}{3}\%$ .

(e) \$975; 40%, 20% and 2%.

2. After a discount of 15% had been deducted, a merchant paid \$850 for a bill of goods; what was the amount of the discount?

3. A merchant paid \$170 for a bill of goods, after a discount of \$30 had been taken off; what was the rate of discount?

4. A trader bought a lot of paper listed at \$5 per ream, at a discount of  $12\frac{1}{2}\%$ ; he received a further reduction of 2% for cash; what did the paper cost him per ream?

5. At what price must an article which cost \$12 be marked, in order that after a discount of 10% has been taken off, there may be a gain of \$3?

6. An article sold for \$4; a discount of  $12\frac{1}{2}\%$  had been given; what was the marked price?

7. A tradesman increases the price of an article which now sells for \$3, so that after giving a discount of 20% he may still receive the former price; find the marked price of the article.

8. A tradesman reduces the price of his goods by 10%; he allows a customer a further reduction of 10% off this latter price; what per cent. of the original price does the customer pay?

9. What is the difference between 20% off, and 10 and 10% off—the marked price being \$12?

10. A merchant marks his goods at an advance of  $33\frac{1}{3}\%$  on cost, and allows a discount of 5% for cash; find the cost of an article of which the cash price is \$7.60.

11. A trader gains \$2.60 after allowing a discount of 10% from the marked price of an article; he marked his goods at an advance of 40% on cost; find the cost.

12. What is the cash value of 150 axes at \$2.50 per dozen, there being a discount of 10% for cash?

13. What is the marked price of a book for which 30 cents is paid, there being a discount of  $33\frac{1}{3}\%$ ?

14. A lot of goods was marked 40% above cost; 30% discount was given; find the gain or loss per cent. of cost.

15. What rate per cent. discount is equivalent to giving one-half of an ounce with each pound for good weight?

16. What rate per cent. discount is equivalent to giving one-half of an inch with each yard for good measure?

17. A bookseller gives a discount of 10% for cash, and allows teachers a second discount of 5% on all cash prices. A teacher paid \$3.42 for a book; what was the marked price?

18. A grocer mixes one quart of water with every two gallons of vinegar; what trade discount will this enable him to give?

19. Suppose you were offered a single discount of 45%, or two discounts of 30% and 20%, which would you take? What would be the difference on merchandise listed at \$1000?

20. What discount is given when 15 articles at the reduced price can be bought for that sum which would pay for 12 articles at the list price?

21. A tradesman makes a gain of 10% of cost after giving a discount of 10%; what would be his gain per cent. of cost if he gave a discount of only 5%?

22. 20 and 10% off is equivalent to what single discount?

23. 10 and what % off is equivalent to 15% off?

24. The marked price of certain goods was reduced on account of damage by fire; a further reduction of 10% is given for cash; goods that were originally marked \$5, were sold for cash for \$3.60; what reduction of marked price was made?

25. Marking goods 5% above cost and giving a discount of 5% off the marked price, is equivalent to what loss per cent. of cost?

26. A merchant offered a reduction of 50 cents from the marked price of a hat; he gave a further reduction of 45 cents for cash, this being at the same rate as the former; what was the selling price of the hat?

27. At what price must an article which cost \$1.50 be marked, so that there may be a gain of 25 cents after a discount of 25 per cent. has been given?

28. Five per cent. of the selling price is equal to four per cent. of the list price; the discount is 10 cents; find the list price.

29. A bill of goods amounting to \$327.50 was bought on Oct. 1, on the following terms: 3 months' credit; or 5% off if paid in 60 days; or 10% off if paid in 30 days. How much would settle the bill on Jan. 1? On Nov. 20? On Oct. 27?

## PROFIT AND LOSS

A gain or loss in business is usually stated as a per cent. of the **Cost Price**.

Thus, if a dealer buys an article for \$200 and sells it at a gain of 15%,

the selling price =  $\$200 + \frac{15}{100}$  of  $\$200 = \$230$ ,

or the selling price = 115% of  $\$200 = \frac{115}{100}$  of  $\$200 = \$230$ .

If he sells it at a loss of 15%,

the selling price =  $\$200 - \frac{15}{100}$  of  $\$200 = \$170$ ,

or the selling price = 85% of  $\$200 = \frac{85}{100}$  of  $\$200 = \$170$ .

## EXERCISE

1. Find the selling price of an article, the cost price and rate of profit or loss being, in each case, as follows:

(a) \$240; 10% profit. (b) \$312.40; 25% loss.

(c) \$73.20;  $12\frac{1}{2}$ % loss. (d) \$210.30;  $16\frac{2}{3}$ % profit.

(e) \$1262.60; 15% profit. (f) \$4562.19;  $11\frac{1}{9}$ % loss.

2. Find the rate per cent. of gain or loss, the cost price and selling price being, in each case, as follows:

- (a) Cost price, \$250; selling price, \$300.
- (b) Cost price, \$210; selling price, \$140.
- (c) Cost price, \$312.40; selling price, \$234.30.
- (d) Cost price, \$1234.56; selling price, \$1440.32.

3. Find the cost price when the selling price and rate of profit or loss are as follows:

- (a) \$630,  $12\frac{1}{2}\%$  loss.      (b) \$630,  $12\frac{1}{2}\%$  gain.
- (c) \$4050, 35% gain.      (d) \$7105, 2% loss.

4. For how much each must hats be sold that cost \$21 a dozen, to gain 20%?

5. A merchant started in business with a capital of \$6000 and gained  $12\frac{1}{2}\%$  the first year and added 75% of the gain to the capital; the second year he gained 20% of this increased capital and added one-half of it to his capital: the third year he gained 40%; what was the amount of gain in the third year?

6. A drover bought 240 sheep at \$8 each; 5% of the flock died; for how much each must he sell the remainder to gain 20%?

7. Twelve hundred reams of paper were bought for \$1000; for what must it be sold per quire to gain  $12\frac{1}{2}\%$ ?

8. A merchant bought a quantity of vinegar at 20 cents a gallon;  $12\frac{1}{2}\%$  leaked away; for how much per gallon should he sell the remainder that he may neither gain nor lose?

9. A bought goods for \$12600; he sells  $\frac{3}{4}$  at a gain of 20%, and the remainder at a loss of 10%; does he gain or lose on the whole?

10. A newsboy buys 12 *Telegrams* for 7 cents; he sells at 1 cent each; what per cent. does he make?

11. If  $\frac{3}{4}$  of a quantity of goods be sold for what the whole cost, what is the gain per cent.?

12. A wine merchant loses 20% of his goods by leakage; by what per cent. must he increase the cost price per gallon to gain 20% on his investment?

13. Sold flour at \$2.70 per hundred, losing 10%; what would be the loss per cent. if sold for \$2.55?

14. If 15% is lost when an article is sold for \$4.25, for what should it be sold to gain 15%?

15. If 5% more is gained when an article is sold for \$4.70 than when sold for \$4.50; find the gain % when the article is sold for \$4.25.

16. Bought wheat and sold so as to gain  $12\frac{1}{2}\%$ ; reinvested the whole sum and made the same rate of gain; reinvested and lost 25%; find my total gain or loss per cent.

17. A drover bought a number of sheep at \$8 each; 10% of the number died; for how much each must he sell the remainder to gain \$1 for each sheep bought?

18. A ship loaded with 4000 bbl. of apples lost 10% of the cargo in a storm; at what advance per cent. of cost must the apples be sold that there may be neither loss nor gain?

19. A dealer sold two horses for \$198 each; on the one he gained 10% and on the other he lost 10%; find his total loss per cent.

20. Bought paper at \$6 per ream; what must I ask per quire for it, so that after throwing off 5%, I may make a gain of 20%?

21. When milk is sold at the rate of 16 quarts for one dollar, there is a gain of 25%; what would be the gain per cent. if 15 quarts were sold for the same sum?

22. If  $\frac{3}{4}$  of the cost price equals  $\frac{2}{3}$  of the selling price, find the gain per cent.

23. A merchant buys 4 gal. 2 qt. of vinegar for one dollar, and sells 2 gal. 3 qt. for the same sum; what does he gain per cent.?

24. Whisky is worth \$1.25 per gallon; how much water must be mixed with each gallon of it, so that when the mixture is sold at \$1.20 per gallon, there may be a gain of  $14\frac{2}{7}\%$ ?

25. What does a grocer gain per cent. by selling  $15\frac{3}{4}$  oz. for 1 pound?

26. A newsboy buys papers at the rate of 8 for 5 cents, and sells at 2 cents each; what profit per cent. does he make?

27. A man buys a horse for \$180; for how much must it be sold to make a profit of 25%, after paying ten dollars to the agent who sells the horse?

28. An apple-woman buys apples at the rate of 100 for 25 cents; 10% of her stock is lost by decay; what per cent. does she gain by selling 5 for 3 cents?

29. A sells a piano to B at a gain of 15%; B sells to C at a gain of 20%; C buys for \$190 more than A paid; what did the piano cost A?

30. A sold an article to B, B sold to C, and C to D; each made a gain of 5%; C's gain was \$5 more than A's; what did D pay for the article?

31. When potatoes were sold for 120% of cost, the gain on each bushel was 8 cents; what was the selling price per bushel?

32. A drover sold 5 cows for \$60 each; on one he gained 10%; on each of two others 25%; on the remaining two he lost  $33\frac{1}{3}\%$ ; did he gain or lose? What per cent.?

33. How many bushels of corn at 40 cents a bushel must be mixed with 100 bushels of oats at 50 cents a bushel, to make a gain of 10% when the mixture is sold at 50 cents a bushel?

34. If 3 articles are sold for the amount that 4 cost, what is the gain per cent.?

35. A and B invested equal sums of money in business; A gained 20% on his money, B gained \$100; B's money was then double A's; what sum did each invest?

36. A grocer sold goods for \$16.10, gaining 15%; if he had sold them for \$20, what would have been his rate of gain?

37. A merchant bought 5 hogsheads (63 gallons each) of molasses at 25 cents a gallon. He paid \$8.20 for freight and \$1.80 for cartage. He loses 10% of the molasses by leakage; 6% of all sales cannot be collected; for how much per gallon must he sell to make a net gain of 40 per cent.?

38. A person buys an article and sells it so as to gain 5%. If he had bought it at 5% less, and sold for \$1 less, he would have gained 10%. Find the cost price.

39. Smith having lost 25% of his capital, is now worth as much as Jones who has just gained 20% of his capital. Jones' original capital was \$9600; what was Smith's original capital?

40. An article costs \$4 and is sold at a gain of 120%; what is the selling price? Could it have been sold at a loss of 120%?

## COMMISSION

If an agent buys or sells goods for another person, the amount he is paid for his services is called **Commission**.

The commission is usually calculated at a certain rate per cent. of the **Buying Price** when the goods are bought, or of the **Selling Price** when the goods are sold.

Thus, if an agent sells goods for \$250 and charges 4% commission, the amount of his commission is 4% of \$250 or \$10,  $\therefore$  the **Net Proceeds**, or the amount remitted to his employer, from the sale of the goods will be \$240.

If an agent buys \$500 worth of goods on a commission of 3%, the commission will be 3% of \$500 or \$15,  $\therefore$  the **Total Cost** of the goods, to his employer, will be \$515.

## EXERCISE

1. Find the commission and the net proceeds of the following sales:

- (a) 2000 bus. of wheat @ 90c. at 3% commission.
- (b) 1200 bus. of potatoes @ 80c. at  $3\frac{1}{2}\%$  commission.
- (c) 120 bbl. of apples @ \$1.60 at 5% commission.
- (d) 300 bales of cotton @ \$42 at  $1\frac{1}{2}\%$  commission.
- (e) 2400 lb. of rice @  $5\frac{1}{2}$ c. at  $6\frac{1}{4}\%$  commission.
- (f) 225 tons of hay @ \$16.20 at 5% commission.
- (g) 1250 doz. of eggs @ 24c. at  $2\frac{1}{2}\%$  commission.

2. An agent sold 15000 bushels of potatoes at 40 cents per bushel; his commission was  $2\frac{1}{2}\%$ ; what sum did he send to his employer?

3. An agent received a consignment of wheat which he sold on a commission of  $1\frac{1}{2}\%$ ; the amount sent to his employer was equivalent to  $65\frac{2}{3}$  cents a bushel; at what price was the wheat sold?

4. An agent transmitted to his employer \$1568, retaining \$32 as his commission for effecting a sale; at what rate was the commission calculated?
5. A commission merchant retained \$40 from the proceeds of sale of 5000 lb. of butter at 20 cents per pound; what rate of commission did he charge?
6. An agent sold a consignment of goods for \$5200 and sent \$5096 to his employer; what rate of commission did he charge?
7. An agent's commission was \$50, for selling some land, at \$125 per acre; how many acres were sold at that price, the commission being reckoned at  $\frac{1}{2}\%$ ?
8. An agent charged \$75 for selling a house for \$10000; what rate of commission was charged?
9. An agent buys for his employer 1000 bushels of wheat at 65 cents a bushel; what is the agent's commission at the rate of  $2\frac{1}{2}\%$ ?
10. How much money must be sent to an agent, that he may purchase a horse for me at \$100, and have his commission of 2%?
11. A merchant sent \$525 to an agent in Atlanta, requesting him to purchase cotton, after retaining his commission of 5% of the sum paid by him for the cotton; what was the commission?
12. \$1750 includes the price paid by an agent for certain goods and his commission of 2%; what sum was paid by the agent for the goods?
13. An agent received \$3570 to be used in buying sugar at 5 cents a pound after paying his commission of 2% of the amount paid by him for the sugar; what was his commission?
14. A commission agent retained \$80 out of \$2480 sent to him for investment and invested the balance; find his rate of commission.
15. I instruct my lawyer to collect an account of \$1250. He collects 80% of it and charges a commission of  $5\frac{1}{2}\%$  on the part collected. How much should he pay me?
16. Sent \$1224 to an agent, requesting him to buy pork for me at \$4.75 per cwt.; he charged 2% commission; how many lb. of pork did he send me?

17. An agent bought 12000 bushels of wheat at 80 cents, and charges  $1\frac{1}{4}\%$  commission; how much money must his employer remit him?

18. A commission agent received a consignment of 638 bbl. of apples; he sold at \$2.50 per bbl., and after deducting the proper commissions, invested the balance in sugar at 4c. per lb. He charged 2% commission on sales, and  $1\frac{1}{2}\%$  on purchases. What was his total commission, and how much sugar did he send to his employer?

Solution.—The selling price of the apples=\$1595.

The commission for selling = \$31.90.

∴ the net proceeds = \$1563.10.

For each 100 lb. of sugar bought the agent pays \$4.

The commission for buying 100 lb.=6c.

∴ each 100 lb. will cost the employer \$4.06,

∴ the number of 100 lb. bought =  $\frac{1563.10}{4.06} = 385$ .

∴ the commission for buying = 6c.  $\times$  385 = \$23.10,

∴ the total commission = \$55.

19. An agent sold on a commission of  $\frac{1}{2}\%$  a cargo of 1200 tons of coal, at \$4.75 per ton; he invested the net proceeds, after retaining a commission of 1%, in lumber at \$18 per M. How many feet of lumber did he buy?

20. A commission merchant received a consignment of 2000 bbl. of flour, which he sold at \$8 a bbl., on a commission of  $1\frac{1}{2}\%$ ; the expenses for freight, paid by the agent out of the proceeds, amounted to \$647.50; he bought cotton at 15 cents a pound with the net proceeds, charging  $\frac{3}{4}\%$  commission for buying. How many pounds of cotton did he buy?

21. A consignment of goods was sold for \$12500; the agent paid \$200 for freight and remitted his employer \$12150. What rate of commission was charged?

22. An agent receives \$6360, with instructions to invest in sugar at 5 cents a pound, retaining his commission at 2% and paying in advance the freight at 20c. per cwt. How much sugar does he buy?

23. An agent receives 1500 hams, average weight 25 pounds, which he sells at 10c. a pound; he pays freight 20 cents per cwt., and charges a commission of 2% on sales. He is instructed to buy tea at 45 cents a pound, to prepay the freight on the tea (20 cents per cwt.), and retain his commission of  $1\frac{1}{2}\%$  on the purchase. How many pounds of tea did he buy?

24. An agent charges  $2\frac{1}{2}\%$  commission on sales and 2% for guaranteeing payment; the sales amount to \$1200. Find the amount the agent receives.

25. An agent charges 2% commission on sales and  $2\frac{3}{4}\%$  for guaranteeing payment; he received altogether \$380. What was the amount of the sales?

26. An agent charges 2% commission on sales and  $2\frac{3}{4}\%$  for guaranteeing payment; his commission for selling was \$40 less than the guaranty commission. What was the amount of the sales?

27. Sold cotton on commission of 4%, invested the net proceeds in sugar at  $1\frac{1}{2}\%$  commission; the total commission was \$220. Find the value of the cotton.

28. An agent received a consignment of wheat, which he sold, charging 2% commission. With the net proceeds, after deducting his commission at  $1\frac{1}{2}\%$ , and prepaying freight at 25 cents per cwt., he bought sugar at  $4\frac{3}{4}$  cents a pound. The agent's total commission was \$70. Find the number of pounds of sugar bought.

29. An agent charges the same rate of commission for buying and selling. He sells a consignment for \$4060, and after deducting \$120 for the two commissions, invests the remainder. Find the rate per cent. charged.

30. A commission merchant charges twice the rate of commission for selling that he does for buying. He sells a consignment of leather for \$3030, deducts as total commission \$90, and invests the balance in hides. Find the rates of commission charged.

Solution.—Commission for selling is calculated on \$3030.

Commission for buying is calculated on \$2940.

Commission on \$3030 at double second rate is the same as commission on \$6060 at second rate.

∴ \$90 is commission on \$6060 + \$2940, or \$9000 at second rate,

∴ second rate is 1%.

31. An agent's rate of commission for selling is one half more than his rate for buying. He sold a consignment of flour for \$4040, and after deducting \$100 for both commissions invested the remainder in tea. What were the rates charged?

32. An agent's rate of commission for selling is  $\frac{4}{3}$  of his rate for buying. He sold a consignment for \$1421, and after deducting \$49 invested the balance. What did he charge for selling?

33. An agent charges \$1 more for selling goods for \$100 than for buying for \$100. He sold a consignment of pork for \$1734, deducted \$85 and invested the remainder in oats. What rate did he charge for investment?

34. An agent sold a consignment of goods, and took \$81 as his commission; he used the remainder in buying goods, deducting \$79 for commission. If his rate for selling is the same as the rate for buying, what is that rate?

## INSURANCE

An Insurance Company, on receiving a certain sum of money, will guarantee to reimburse the owner of property, in case it should be destroyed or damaged by fire.

The contract between the owner and the company is called the **Policy**.

The sum paid by the owner to the company is called the **Premium**.

The sum that the company guarantees to pay to the owner is called the **Face of the Policy** or **Risk**.

The amount of premium to be paid is generally stated as a certain per cent. of the risk, or as a certain sum on each \$100 of risk.

Houses and their contents are usually insured for three years, business properties for one year.

Thus, if a store is worth \$10000 and is insured for \$6000, the rate being 2%, the premium paid by the owner to the insurance company is 2% of \$6000, or \$120.

If the store is completely destroyed the company will pay the owner \$6000. If it is damaged to the extent of \$2000, the company will pay the \$2000. In any case the company will pay the estimated amount of the damage, if it is not more than \$6000.

Irrespective of the amount of insurance on property which has been partially or totally destroyed, the insurance company is liable for the actual amount of loss only, provided that loss is not more than the sum for which the property was insured.

When the same property is insured in different companies, any loss which occurs is paid by them in proportion to their risks, without regard to the premiums they have respectively received.

If property which is insured for \$6000 is damaged to the extent of \$2000, which is paid by the company, the risk on the property for the remainder of the insurance term will be only \$4000 under that policy.

If the furniture in a house is insured for \$1200 for 3 years, the rate being 65c. per \$100, the premium for the 3 years is  $65c. \times 12$  or \$7.80.

There are other kinds of insurance such as marine, life, accident, etc.

In marine insurance the rate of premium is usually stated as a per cent. of the risk.

In life and accident insurance the rate is usually stated as a certain sum for each \$1000 of insurance.

Thus, the annual premium to be paid by a person 21 years of age, for a straight life policy, when insured in many of the Canadian companies, is \$19.40 per \$1000.

### EXERCISE

1. Find the premium to be paid for each of the following risks at the rate specified:

- |                                |                                 |
|--------------------------------|---------------------------------|
| (a) \$2000, $1\frac{1}{2}\%$ . | (b) \$4500, $2\frac{1}{2}\%$ .  |
| (c) \$640, $4\frac{1}{4}\%$ .  | (d) \$20500, $1\frac{1}{5}\%$ . |
| (e) \$7500, 80c.               | (f) \$1800, 63c.                |

2. Find the risk if the premium and rate are, in each case, as follows:

- |                                |                                 |
|--------------------------------|---------------------------------|
| (a) \$12.50, 2%.               | (b) \$37.50, $1\frac{1}{2}\%$ . |
| (c) \$22.25, $\frac{1}{2}\%$ . | (d) \$38.50, 70c.               |

3. Find the rate, in each case, when the risk and premium are as follows:

- |                      |                      |
|----------------------|----------------------|
| (a) \$2500, \$31.25. | (b) \$6500, \$48.75. |
| (c) \$1200, \$6.60.  | (d) \$1650, \$8.91.  |

4. An insurance company charges  $\frac{7}{8}\%$  per annum; what is the premium paid on a policy of \$4000, in force 3 years?

5. What premium is paid for a policy of \$1800, in force 5 years, the rate of insurance being  $\frac{5}{8}\%$  of the policy for each year?

6. Find the premium paid to insure a house worth \$12000, for  $\frac{3}{4}$  of its value, for 3 years, the rate being  $\frac{2}{3}\%$  of the policy for each year.

7. A premium of \$60 is paid an insurance company for two years' insurance, the amount of the policy being \$4000; what is the yearly rate?

8. \$50 is paid to secure a policy of \$2500 on a house, to run 3 years; what is the yearly rate?

9. \$42 is paid to insure for two thirds of its value, a house worth \$6000; the policy is to run for 2 years; what is the yearly rate?

10. A cargo worth \$1250 is insured for 75% of its value; the premium paid was \$12.50. Find the rate.

11. A house was insured for 3 years by paying a premium of \$24; the rate was  $\frac{1}{2}\%$  a year; find the value of the house,  $\frac{3}{4}$  of its value being insured.

12. A premium of \$37.50 was paid for a two-year policy on a dwelling worth \$6000; the rate was  $\frac{1}{2}\%$  of the policy for each year; what fraction of the value was insured?

13. An insurance company charged \$18.45 for insuring a house for \$2460, for one year; find the rate per cent.

14. An insurance company took a risk for one year of \$10000 on a warehouse worth \$15500, at  $\frac{3}{4}\%$ ; it covered 40% of its risk in another company at  $\frac{7}{8}\%$ ; how much premium did the first company receive above that which it paid the second?

15. A house is worth \$4950; for how much must it be insured at 1%, so that in case of loss the owner may recover 80% of the value in addition to the premium paid?

Solution.—The face of the policy = 80% of the value + the premium,  
 $\therefore$  the face of the policy = \$3960 + 1% of the policy,  
 $\therefore$  99% of the face of the policy = \$3960,  
 $\therefore$  the face of the policy = \$4000.

16. For how much must a cargo worth \$7940 be insured at  $\frac{3}{4}\%$ , so that the owner suffers no loss if the cargo is lost?

17. For what sum should a house worth \$7930 be insured to cover  $\frac{1}{2}$  of the value of the house, and the cost of the policy at  $\frac{7}{8}\%$ ?

18. For what sum must a vessel worth \$15800 be insured to cover the value of the vessel, the cost of the policy at  $\frac{5}{8}\%$ , and \$100 besides?

19. What must be the amount of the policy taken on a cargo worth \$5940 to cover the value of the cargo, the premium paid, and an additional sum equal to the premium, the rate being  $\frac{1}{2}\%$ ?

20. A company took a risk of \$40000 at  $1\frac{3}{4}\%$ , re-insured 40 per cent of it at 2% and 25 per cent. at  $2\frac{1}{4}\%$ . What rate of insurance did the company get on the amount of risk it retained?

21. A company took a risk at  $1\frac{1}{2}\%$ , re-insured 50% of it at  $1\frac{3}{4}\%$ , and 20% of the remainder at  $1\frac{1}{4}\%$ . What rate did the company receive on the amount of risk it carried?

22. A block of buildings worth \$1000000 was insured in company No. I for \$25000 at  $1\frac{1}{4}\%$ ; in company No. II for \$40000 at  $1\%$ ; in company No. III for \$100000 at  $\frac{7}{8}\%$ . Find the premiums paid in each case. If the block be damaged to the extent of \$100000 what amount of loss will be borne by each company?

23. A house is insured for  $\frac{3}{4}$  of its value, the furniture for  $\frac{2}{3}$  of its value. The rate in both cases is  $\frac{3}{4}\%$ . The house is worth 5 times as much as the furniture; the total premium paid is \$13.25. Find the value of the house.

24. The premium on a vessel and its cargo is \$120; the rate on the vessel is  $\frac{1}{2}\%$  and on the cargo  $\frac{3}{8}\%$ . The value of the cargo is double that of the vessel, and each is insured for  $\frac{3}{4}$  of its value. Find the value of the cargo.

25. A vessel was insured for \$20000 at  $\frac{3}{4}\%$  in one company, and for \$25000 in another at  $\frac{3}{8}\%$ . What rate of premium is paid on the whole insurance?

26. For what sum must a vessel worth \$18000 and cargo worth \$24000 be insured at  $\frac{3}{4}$  and  $\frac{1}{2}\%$  respectively, to cover the total value and the premiums paid?

27. A merchant had 500 barrels of flour insured for  $\frac{3}{4}$  of its value at  $2\frac{1}{2}\%$ , paying \$75 premium. At what price per barrel must he sell to gain  $25\%$  of cost, as well as premium paid?

28. What is the value of a house if the insurance premium at  $\frac{3}{5}\%$  on  $\frac{3}{4}$  of its value, including 50 cents for the policy, equals \$21.50?

29. A man built a house, costing \$2500, upon a lot worth \$500; the house was burned and the insurance company paid the full amount of the policy,  $\frac{3}{5}$  of the value at  $\frac{3}{5}\%$ ; the land was then sold for \$750. What was the man's total gain or loss?

30. A man insures his house so that in case of loss he may recover the value of the house and the premium of insurance at  $1\frac{1}{4}\%$ . The house is destroyed by fire and  $\frac{4}{5}$  of the claim is allowed; he finds that he receives \$750 less than the value of the house. Find the value of the house and the premium.

31. A house worth \$5935 is insured for \$3000 to cover  $\frac{1}{2}$  of the value and the premium paid; what was the premium paid?

32. A vessel worth \$4925 is insured for \$4000, which sum includes  $\frac{4}{5}$  of the value of the vessel, and the premium paid; what was the rate of insurance?

33. A cargo worth \$22125 is insured for \$15000, to cover  $\frac{2}{3}$  of the value, the premium, and \$100 besides; what was the rate of insurance?

34. Company No. I insured a building and its stock for  $\frac{2}{3}$  of the value, charging  $1\frac{3}{4}\%$ . They re-insured in Company No. II  $\frac{1}{4}$  of the risk at  $1\frac{1}{2}\%$ ; building and stock being destroyed by fire, the second company lost \$49000 less than the first. What amount did the owners lose?

35. My house is valued at one-half more than my brother's; my house is insured at  $\frac{3}{4}\%$  on  $\frac{5}{8}$  of its value, my brother's at  $\frac{5}{8}\%$  on  $\frac{2}{3}$  of its value; I pay \$12 more premium than my brother; find the value of each house.

36. A shipment of flour was insured at  $\frac{1}{4}\%$ , to cover  $\frac{1}{2}$  of the value and the premium; the premium was \$15; find the value of the flour.

37. A drover is taking a herd of 400 cattle from Quebec to Liverpool; the average cost of the cattle was \$45; for what sum must he have the cattle insured at  $1\frac{1}{4}\%$ , to cover, in case of loss, the value of the cattle, the premium paid, and the cost of his passage, \$84?

## TAXES

Sums of money collected for the purpose of paying the expenses of a municipality are called **Taxes**.

The amount of money required to meet these expenses for the current year, as well as the value of all the taxable property and income, are determined by the council of the municipality.

The value of the taxable property and income is called the **Assessment**.

From the assessment and the amount of estimated expenditure the **Rate of Taxation** is obtained.

Thus, if the estimated expenses of a city for a certain year are \$600,000, and the assessment is \$30,000,000, the taxes on \$30,000,000 will be \$600,000,

$\therefore$  the taxes on \$1 will be  $\$ \frac{600,000}{30,000,000}$  or \$.02.

$\therefore$  the rate of taxation is 2%, 2c. on the dollar, or 20 mills on the dollar.

The rate of taxation is usually expressed as a number of mills on the dollar of the assessment, or as a rate per cent. of the assessment.

If the rate of taxation is 15 mills on the dollar, the taxes would be  $\frac{15}{1000}$  of the assessment, so that the taxes on property or income assessed at \$1000 would be \$15.

If the assessment is \$3000 and the rate is 19 mills, the taxes on \$1000 = \$19,

$\therefore$  the taxes on \$3000 =  $\$19 \times 3 = \$57$ ;

or, the taxes on \$3000 =  $\$3000 \times \frac{19}{1000}$   
 $= \$3000 \times .019 = \$57.$

If on \$3500 the taxes = \$56,

$\therefore$  on \$1000 the taxes =  $\frac{56}{3500}$  of \$56 = \$16,

$\therefore$  the rate is \$16 on \$1000, or 16 mills on the dollar.

A person does not pay taxes on all of his income. The part on which he does not pay is said to be **exempted** from taxation.

Thus, if a person's income is \$1800 a year, of which \$1000 is exempted, he would pay taxes on only \$800.

## EXERCISE

1. Every citizen of Borden is called upon to pay for public use 2% of the value of his property. What tax does Jones pay whose property is worth \$5000?

2. My property is worth, according to the assessor, \$2500. The rate of taxation is  $1\frac{1}{2}$  cents on the dollar. What is the amount of my taxes?

3. Find the tax on property assessed at \$12000, when the rate of taxation is 2 cents on the dollar.

4. When the rate of taxation is 15 mills on the dollar, what is the tax on property assessed at \$2000?

5. The assessed value of the property in a town is \$750,000. What tax will be raised when the rate is  $12\frac{1}{2}$  mills on the dollar?

6. A city requires for the expenses of one year the sum of \$1,500,000; the taxable property of the corporation is assessed at 135 million dollars. Find the rate of taxation.

7. The property of a village is assessed at \$800000; the rate of taxation is 18 mills on the dollar; it costs 2% of the tax for collection. Find the net amount received by the village.

8. A town requires \$19600 to meet expenses for the year. 2% is paid for collection. What must be the rate, if the taxable property is assessed at \$1,200,000?

9. A's income is \$1200 annually, of which \$400 is exempted from taxation. What tax does he pay when the rate is  $1\frac{2}{3}$ %?

10. What income tax does a man pay whose income is \$1500 a year, the rate being 16 mills on the dollar; \$500 being exempted from taxation?

11. What is my net income when I receive a salary of \$2000 a year, \$600 of which is exempted, the rate being 15 mills on the dollar?

12. My salary is \$1500; my net income is \$1482.40 after paying income tax on all over \$400. What is the rate?

13. Mr. Jones' annual income is 25% of his capital; he pays \$28 taxes, at the rate of  $1\frac{1}{4}$ % on income. What is his capital?

14. My salary is \$1800; my net income is \$1779 after paying income tax on all over \$400. What was the rate?

15. The expense of constructing a bridge was \$10000, which was raised by a tax on the assessable property of a town. The rate of taxation was 2%, and the collector's commission was \$150. Find the assessed value of the property of the town.

16. Incomes of \$1000 or more pay tax on all over \$400; incomes less than \$1000 are exempted from taxation; the rate of taxation is 15 mills on the dollar. How much better off is a man whose income is \$995 than a man whose income is \$1000?

17. A tax of \$4250 is levied on a village of which the assessed value of the property is \$255000. What is the tax on property valued at \$1800?

18. A tax of \$15000 is levied on a town, the assessed valuation being \$930000. What tax does a man pay whose income is \$1300, \$400 of which is exempted?

19. I paid \$24 income tax, \$400 of my income being exempted; the rate was 16 mills on the dollar. What was my income?

20. Smith bought a house for \$6000; it is assessed for  $\frac{2}{3}$  of its value, the rate of taxation being  $16\frac{1}{2}$  mills on the dollar. The house is insured for a year for  $\frac{3}{4}$  of its value at  $\frac{1}{2}\%$ . If Smith had loaned his money he might have received \$300 interest for the year. What monthly rent is Smith really paying for his house?

21. A tax of \$4500 is levied on a village, the assessed valuation being \$180000. What tax does a man pay whose income is \$1350, \$400 being exempted from taxation?

22. The total assessment of the City of Hamilton in 1908 was \$39,000,000. The taxes were divided as follows:

Debenture rate.....	3.4	mills.
City rate.....	9.9	"
Public School rate.....	4.6	"
Collegiate Institute rate.....	1.1	"
Free Library rate.....	.5	"
Park Fund rate.....	.5	"

Find the total taxes and the amount realized by each of the foregoing rates.

## DUTIES AND CUSTOMS

The income of the Government of Canada consists chiefly of taxes levied upon goods brought into the country. This income is called a **Customs Revenue or Duty**.

The Government also receives a large revenue from a tax upon certain goods, chiefly liquors and tobacco, which are manufactured in Canada. This is called an **Inland Revenue or Excise Duty**.

These taxes or duties are of two kinds, **Specific** and **Ad Valorem**.

A specific duty is assessed upon the number, weight or measure of the goods, without regard to value.

An ad valorem duty is calculated as a certain per cent. of the value of the goods at the place of purchase, that is, of the **Invoice Value**.

Certain goods imported from Great Britain are subject to a **Preferential Tariff**, which is lower than the duty on the same goods when imported from other countries.

Thus, the present duty on gloves from Great Britain is  $22\frac{1}{2}\%$ , and from other countries is  $35\%$ .

The following is a tariff of certain dutiable articles, the first rate being the preferential, the second the general:

Barley, per bu.....	10c.	15c.
Blankets.....	30%	35%.
Flour, per bbl.....	40c.	60c.
Iron plates.....	free	5%.
Plate glass.....	$7\frac{1}{2}\%$	10%.

Prunes, per lb.....	$\frac{1}{2}$ c.	1c.
Salt, per 100 lb.....	free	$7\frac{1}{2}$ c.
Sugar, per 100 lb....	40c.	$62\frac{1}{2}$ c.
Wall paper.....	$22\frac{1}{2}\%$	$35\%$ .

## EXERCISE

1. Find the duty on each of the following when imported: (a) from Great Britain, (b) from other countries.

- 1200 bu. of barley.
- A car load of sugar weighing 10 tons.
- 1200 boxes of prunes, each weighing 25 lb.
- A shipment of iron plates valued at \$650.
- 1000 rolls of wall paper costing 12c. a roll.
- 200 barrels of salt.
- 125 pieces of plate glass worth \$12.40 each.
- A bale of blankets worth \$340.60.
- 49 tons of flour.

2. During the year 1906 the value of goods imported into Canada was 290 million dollars, the duty collected was  $46\frac{1}{2}$  million dollars. What was the average rate of duty?

3. A dealer imports for me a book which was invoiced to him at \$2.40; he pays 20 cents postage, 15% ad valorem duty, and makes a gain of 25% on the whole outlay. What does he charge for the book?

4. If there were no duty in the previous problem, what would the dealer charge me for the book so as to make the same *rate* of gain? What would be the charge to make the same *amount* of gain?

5. I import a piano on which there is a specific duty of \$30 and an ad valorem duty of 20 per cent.; I pay altogether for the piano \$390. What was the invoice price?

6. Find the duty paid on a hogshead of molasses, invoiced at 40 cents a gallon, at 15 per cent. ad valorem.

7. What reduction, per gallon, might be made in the price of the molasses, in the previous problem, if there were no duty, the dealer selling at 25% above total cost?

8. Find the export duty on a pine log of uniform section, the length being 30 ft., the diameter 2 ft. 11 in., and the rate of duty \$1.50 per cord.

9. Find the export duty on a stick of timber 20 ft. long, 3 ft. broad and 2 ft. thick, the rate of duty being \$2 per 1000 feet board measure.

10. The duty paid on a consignment of 50 pounds of manufactured tobacco was \$17.50; the duty on such tobacco is 30 cents a pound and  $12\frac{1}{2}\%$  on the value. Find the value of the tobacco as shown in the invoice.

11. A dealer in pianos sells at an advance of 40% on the cost, laid down in his store. I pay him \$304 for a piano, on which he paid a specific duty of \$30 and an ad valorem duty of 15%. What was the invoice price of the piano?

12. Giving the dealer the same rate of profit, by how much would the price be reduced in the preceding problem if there were no duty?

13. A grocer imported 150 cases of port wine, 24 bottles in each case. After 5% had been allowed for breakage, he paid an ad valorem duty of 20%. The freight and cartage expenses were \$100, and the whole cost was \$4384; what was the invoice price per bottle?

14. If goods invoiced at \$1200 cost \$1800 when laid down in warehouse, the cartage and freight amounting to \$75, what was the rate of duty?

15. The duty on surgical instrument cases is 35%; that on the instruments 20%; the duty paid on a case of instruments invoiced at \$30 was \$7.50; find the invoice price of the instruments alone.

16. The duty on 1000 boxes of raisins, each containing 15 pounds, was \$270; the raisins were invoiced at 8c. per lb.; the specific duty being 1 cent a pound, determine the ad valorem duty.

17. The duty on imported axes is \$2 per dozen and 10% ad valorem. The whole duty paid on a lot of axes was \$56, the specific duty being \$24 more than the ad valorem. Find the number of axes imported and the invoice price per doz.

18. The duty on a bale of canton flannel was \$3.75, the specific duty being 1 cent per square yard, and the ad valorem 15%; find the width of the flannel, given that the bale contained 100 yards, invoiced at 20 cents.

19. The duty on imported window shade rollers is 30% ad valorem; on the shades 5 cents per square yard and 15% ad valorem. Each shade is worth twice the value of the roller. The duty paid on a dozen rollers and shades, invoiced at \$3 each, was \$9.90; the width of each shade was 4 ft. 6 in.; find the length.

20. The duty on rubber-lined cotton fire hose is 5 cents per pound and 15% ad valorem. The duty on 100 feet of hose, invoiced at 20 cents per foot, was \$15.50; find its weight per foot.

## PARTNERSHIP

The association of two or more persons with joint capital, for the carrying on of some particular business, is called a **Partnership**.

Thus, if a certain enterprise requires a capital of \$5000, we may suppose that A supplies \$2000 of this sum, B \$1250, and C \$1750.

Any gain or loss which results will naturally be divided among A, B and C in proportion to their capitals.

Thus, if there is a gain of \$1500,

A would receive  $\frac{2000}{5000}$  of \$1500 = \$600,

B would receive  $\frac{1250}{5000}$  of \$1500 = \$375,

C would receive  $\frac{1750}{5000}$  of \$1500 = \$525.

If A and B were partners, A having invested \$600 for 3 months and B \$500 for 4 months,

A would be entitled to receive the gain on \$600 invested for 3 months, or the gain on \$1800 invested for 1 month, while B would receive the gain on \$2000 for 1 month.

The net gain or loss would therefore be divided between them in the ratio of 1800 to 2000 or 9 to 10.

### EXERCISE

1. Jones and Smith engage in business, each furnishing \$5000; at the end of one year they have made a gain of \$2500. How should this gain be divided?

2. Jones invests \$4000 and Smith \$6000 in a joint business; they make a gain of \$2800 in six months. How should this gain be divided?

3. A and B engaged in the lumber trade, with a joint capital of \$12000; at the end of a year A's gain amounted to \$400 and B's to \$300. How much capital did A put in the business?

4. Sykes and Smith formed a partnership, with a joint capital of \$8000—Sykes to receive \$1200 a year for managing the business; the total gain for the year was \$3200, of which Smith received \$1500. What amount of capital did Sykes invest?

5. A, B and D formed a partnership; their respective shares of one year's gain are \$2000, \$3000 and \$5000; A invested \$4000 less than B. How much did D invest?

6. A invested \$4500, for 2 months, in a certain business; B invested \$4000, for 3 months. If the gain is divided in proportion to the use of each man's investment, what amount should A receive out of a total gain of \$2800?

7. A invested \$1600, for 3 months; B \$1100, for 2 months; C \$3000, for  $1\frac{1}{2}$  months; the total gain was \$2400. Find each man's share.

8. A invested \$2400 in a business for 6 months, acting as manager for that time on a yearly salary of \$1200; B invested \$3000 for 4 months, and during that time received \$350 as bookkeeper; C invested \$4000 for 12 months, and acted as manager and bookkeeper when A and B were not in the business; the total gain for the year was \$8640. What was C's share?

9. At the beginning of a year A, B and C enter into a partnership, each contributing \$4000. At the end of 4 months A withdraws one-half of his investment, and at the end of 6 months B withdraws  $\frac{3}{4}$  of his. The gain for the year is \$6000. Find C's share.

10. A, B and C entered into partnership, contributing respectively \$3500, \$2200, and \$2500; their gains were \$1120, \$880, and \$1200 respectively. If B's capital was in trade 2 months longer than A's, for what time was each man's money in the business?

11. A, B and C formed a partnership, their money being in the business for 2 months,  $2\frac{1}{2}$  months and 4 months respectively; their gains were \$600, \$500 and \$800 respectively; A's investment was \$3000. Find B's and C's.

12. Hardy and Jones are in partnership, Hardy having invested \$12000 and Jones \$15000; Hardy acts as manager on a yearly salary of \$2400, the salary to be reduced in proportion if the capital is reduced; at the end of 4 months Hardy takes \$3000 out of the business, and at the end of 6 months Jones takes out \$4000; the total gain for the year is \$6000. How much of this does Hardy receive?

13. Four men form a partnership, the second puts in twice as much capital as the first, the third as much as the first and second, and the fourth as much as the other three. How should a profit of \$4800 be divided among them?

14. B and C formed a partnership to dig a trench; B furnished 100 workmen for 40 days, C 120 workmen for 30 days; they received \$12000 for the work. What was the share of each?

15. A owns  $\frac{1}{4}$  of a vessel, B  $\frac{1}{3}$  and C the remainder; the vessel is insured for  $\frac{3}{4}$  of its value; the vessel is lost, and A, after receiving his share of the insurance, finds that he has lost \$1000. What do B and C lose respectively?

16. At the end of a year, from the commencement of their business, Smith, Jones and Cook, after "taking stock," find the amount of goods on hand to be \$40000; cash on hand, \$22000; debts due them, \$25000; amount of their indebtedness, \$47000.

Make a statement of resources and liabilities, showing net capital and gain. Find each partner's share of the gain, Smith having put in the business \$8000, Jones \$9000 and Cook \$3000.

17. Three men rent a pasture for \$176.25. A puts in 3 horses for 9 weeks; B puts in 7 cows for 6 weeks; and C puts in 30 sheep for 4 weeks. If it costs as much to pasture 2 horses as 3 cows, and 4 cows as 7 sheep, how much ought each to pay?

18. Terry rented a house for one year for \$480; at the end of three months he took in Tucker as a co-tenant; after four months more they admitted Taylor; Tucker moved out one month before the year was up; how much rent did each pay?

## STOCKS

When money is required to carry on a specified business, it may be supplied by different individuals.

These persons, when properly organized as one body, are said to form a **Joint Stock Company or Corporation**.

The money which is necessary to carry on the business is called the **Capital** of the company.

This capital is divided into **Shares**, usually of \$100 each.

Each person who owns one or more of these shares is a **Stockholder or Shareholder** and receives a **Share or Stock Certificate** stating how many shares he owns.

<b>SHARE CERTIFICATE</b>	<b>BANK OF HAMILTON</b>	
	Hamilton, July 10 <sup>th</sup> 69	
(This is to Certify that <u>Norris B. German</u> of <u>Toronto</u> holds at this date <u>Ten</u> Shares in the Capital Stock of the Bank of Hamilton of the value of <u>ONE HUNDRED DOLLARS PER SHARE</u> the said Shares being transferable on the Books of the Bank only by <u>him or</u> <u>his</u> <u>Attorney</u> duly constituted.		
<u>Ten</u> Shares <u>J. K. Jackson</u> General Manager		

These shares may usually be bought and sold.

If the business of the company is prosperous, a share may sell for more than its **Face** or **Par** value, which is \$100.

The stock is then said to be at a **Premium**.

Thus, if a share sells for \$115 it is said to be at \$15 premium or 15% above par. If it sells for \$90 it is said to be at \$10 discount or 10% below par.

The shares are generally bought and sold through agents called **Stock Brokers**.

The broker charges a certain sum per share for buying or selling, and as the shares of most of the large stock companies have a par value of \$100 the customary way of quoting the broker's charge is by a per cent.

The charge made by the broker is called **Brokerage**.

Thus, if the brokerage is  $\frac{1}{8}\%$ , the charge for buying or selling one share would be  $\$ \frac{1}{8}$ .

If I instruct a broker to buy for me 20 shares of C.P.R. stock which is quoted at 165, he would pay \$165 for each share, and would charge me  $\$ \frac{1}{8}$  for buying it,

$\therefore$  each share would cost me  $\$165\frac{1}{8}$ ,

$\therefore$  20 shares would cost me  $\$165\frac{1}{8} \times 20$  or \$3302.50.

The brokerage would be  $\$ \frac{1}{8} \times 20$  or \$2.50.

If I own 20 shares of C.P.R. stock and I instruct a broker to sell them,

the net selling price of each share would be  $\$164\frac{7}{8}$ ,

$\therefore$  20 shares would sell for  $\$164\frac{7}{8} \times 20$  or \$3297.50.

The net profits of a company, which are distributed among the stockholders at stated periods, are called **Dividends**.

If a dividend of 6% is declared it means that each owner of one share will receive in cash 6% of the par value of that share, which is \$6 on each \$100 share.

Thus, if I own 10 shares of a stock which pays a 4% dividend, I shall receive from the company a cheque for \$40 each year if the dividend is paid annually, \$20 each half-year if paid half-yearly, or \$10 every three months if paid quarterly.

The prices of certain Canadian stocks, as reported in the newspapers of May 28th, 1909, were as follows:

Canadian Pacific Railway.....	180
Toronto Street Railway.....	125
Richelieu Navigation Co.....	81½
Dominion Steel, Common.....	38
Dominion Steel, Preferred.....	120
Bank of Hamilton.....	200
Bank of Montreal.....	249
Traders Bank.....	139¾
National Trust Co.....	163
Dominion Coal.....	73¼

### EXERCISE.

1. Find the cost of 10 shares of each of the above named stocks at the price stated.

2. Find the cost of 20 shares of Dominion Coal at the price quoted, if bought through a broker who charges ¼%.

3. Find the net selling price of 25 shares of Traders Bank stock at the price quoted, the brokerage being ¼%.

4. Bank of Hamilton stock pays a dividend of  $2\frac{1}{4}\%$  every three months. If I own 12 shares what quarterly dividend would I receive? How much would I receive in 2 years?

5. If I buy 10 shares of C.P.R. stock at  $173\frac{1}{2}$  and sell at 180, how much do I gain?

6. In question 5, if I employ a broker who charges  $\frac{1}{4}\%$  for buying and the same for selling, how much would I gain?

7. Canadian Pacific Railway stock pays a dividend of 7%. What rate per cent. per annum do I make by investing money in it at the price quoted?

Hint—On each \$180 I invest I receive a yearly dividend of \$7.

8. If I have \$1000 to invest in each of the stocks quoted, what is the largest number of shares of each which I can buy, and how much cash will remain in each case?

9. How much stock must be sold at  $117\frac{3}{4}$ , to produce \$4710?

10. How much stock must be sold to produce \$4710, the stock being quoted at  $117\frac{7}{8}$ , brokerage  $\frac{1}{8}$ ?

11. Find the income received from \$4000 stock (40 shares), paying an annual dividend of 8%.

12. Invested \$9100 in bank stock at  $90\frac{7}{8}$ , brokerage  $\frac{1}{8}$ , and sold out at 92, brokerage  $\frac{1}{8}$ ; what did I gain?

13. Bought 86 shares, quoted at  $96\frac{1}{8}$ , and sold when the stock had fallen to 95; how much did I lose, brokerage  $\frac{1}{8}$  each way?

14. What rate per cent. do I make on my money by investing in stock at 95 which pays a 5% annual dividend?

15. What rate per cent. do I receive on my money by investing in stock at  $94\frac{7}{8}$ , brokerage  $\frac{1}{8}$ , paying an annual dividend of 5%?

16. Find the rate of dividend paid by stock, when a man who owns 240 shares of it receives \$1920.

17. How much 6% stock must be bought to give an annual income of \$240? What will it cost at 75, brokerage  $\frac{1}{4}$ ?

18. What is the price of a 7% stock which pays 5% on the money invested?

19. What is the price of a 5% stock, which pays  $4\frac{1}{2}\%$  on the money invested, brokerage  $\frac{1}{8}$ ?

20. If 500 shares of 6% stock are sold at  $104\frac{1}{8}$  and the proceeds invested in an 8% stock at  $124\frac{1}{8}$ , find the alteration in income, brokerage each way  $\frac{1}{8}$ .

21. A man decreases his income \$480 by selling out of 3% stock at 67 and investing in 4% at par. What amount of 4% stock does he buy?

22. A man sells out of 3% stock at  $67\frac{1}{8}$  and invests in 4% stock at  $99\frac{1}{8}$ ; his income is decreased by \$480. What amount of 3% stock does he sell, brokerage  $\frac{1}{8}$ ?

23. Bought \$4800 stock at 75. At what price, per share, must I sell it to gain \$150?

24. A man receives a half-yearly dividend of 4% on the amount of his stock, and invests it in the same stock at 120. His next half-yearly dividend is \$496. What is the amount of the first dividend?

25. What must be the price of consols in order that after deducting an income tax of 2% an investor may make  $3\frac{1}{2}\%$  on his money, the consols paying 3%?

26. The expense of constructing a railroad was \$4,000,000, of which 40% was borrowed on mortgage at 6%, and the remainder is held in shares. What must be the average weekly receipts to pay the shareholders 5%, the working expenses being 65% of the gross receipts?

27. The yearly gain of a company whose capital stock is one million dollars, is \$55000. What rate of dividend can be declared to ordinary shareholders, after paying 8% on the preference stock, which is one-half of the whole amount?

28. Which is the more profitable investment, 6% stock at 128, or 5% stock at 99, brokerage  $\frac{1}{8}$ ?

29. The whole stock of a company is \$1,000,000, the net gain is sufficient to pay 6% on the whole amount. The company pays 3% to ordinary shareholders, the balance giving 8% on the preference stock. What is the amount of the preference stock?

30. A company with a capital of \$200000 paid 8% dividend to its shareholders. Afterwards new stock was issued, and, with the same amount of gain, the company paid only 5%. What was the amount of the new stock issued?

31. A man invests \$6000 in 3% stock at 75; he sells out at 90, and invests  $\frac{1}{3}$  of the proceeds in  $3\frac{1}{2}\%$  stock at 96, and the remainder in 5% stock at par. Find his income from the latter investments.

32. I sold some stock at a discount of 10% and made  $12\frac{1}{2}\%$  on my money, at what rate of discount did I buy?

33. When money is worth 4%, what ought to be the price of consols which pay 3%?

34. A man invests \$40000 in  $5\frac{1}{2}\%$  stock at  $79\frac{3}{4}$ , and \$60000 in  $7\frac{1}{2}\%$  stock at  $119\frac{3}{4}$ , brokerage in each case  $\frac{1}{4}$ ; what is his total income, and what does the broker receive for his services?

35. A person owns \$15000 bank stock paying 5%, which he sells and invests the proceeds in 6% stock at 120, his income being increased by \$60; find the price at which he sells the first stock.

36. What sum invested in a 5% stock at 115 will yield a net income of \$1779 after paying an income tax of 15 mills on the dollar on all over \$400?

37. How much stock, at 12% discount, must be bought, and sold at 8% discount, to make a clear gain of \$300, brokerage each way  $\frac{1}{8}$ ?

38. A man invested a certain sum of money in a 6% stock at  $119\frac{7}{8}$ , brokerage  $\frac{1}{8}$ , and half as much more in a 5% stock at  $99\frac{7}{8}$ , brokerage  $\frac{1}{8}$ ; his income from the two investments was \$900. How much did he invest in each kind of stock?

39. If 12 shares of a 4% stock are sold at 90, and the proceeds invested at  $4\frac{1}{2}\%$  per annum, find the change in income.

## SIMPLE INTEREST

**Interest** is the sum of money charged for the use of money.

It is usually a certain per cent. per annum of the sum loaned.

The sum loaned is called the **Principal**.

The whole sum due at the end of the time, that is the sum of the principal and interest, is called the **Amount**.

Thus, if I loan \$1250 for  $2\frac{1}{2}$  years at 4%,  
 the interest for each year is  $\frac{4}{100}$  of \$1250 = \$50,  
 $\therefore$  the interest for  $2\frac{1}{2}$  years =  $\$50 \times 2\frac{1}{2} = \$125$ ,  
 $\therefore$  the amount =  $\$1250 + \$125 = \$1375$ .

Find the interest on \$500 from Jan. 5th, 1908, to Mar. 13th, 1908, at 6% per annum.

The number of days from Jan. 5th, 1908, to Mar. 13th, 1908, is  $(26 + 29 + 13)$  or 68.

The interest on \$500 for 1 year = 6% of \$500 or \$30.

$\therefore$  the interest for 68 days

$$= \frac{68}{365} \text{ of } \$30 = \$5.59 \text{ (to the nearest cent).}$$

In finding the interest for the number of days between two given dates in any year, the whole year should be considered as 365 days.

## EXERCISE

1. Find the simple interest on;

- |            |                          |                  |
|------------|--------------------------|------------------|
| (a) \$1200 | for 3 years              | at 4% per annum. |
| (b) \$1750 | for $2\frac{1}{2}$ years | at 3% per annum. |

- (c) \$926.50 for  $2\frac{1}{2}$  years at 6% per annum.
- (d) \$1827.60 for 2 years and 4 mo. at 6% per annum.
- (e) \$925.40 for 4 years and 5 mo. at 4% per annum.
- (f) \$1000 for 2 years and 65 days at 5% per annum.
- (g) \$1263.80 for 3 years and 73 days at 10% per annum.
- (h) £1800 12s. 6d. for 2 years and 146 days at 5% per annum.

2. I loan \$100 for one year at 6%; what sum should be returned to me at the end of the year?

3. Find the amount of \$1200 loaned for three years at 4% per annum.

4. Find the interest on \$1400 loaned on May 1st, 1907, and returned Nov. 14th, 1907, at 8% per annum.

5. Find the interest on \$1650 from Jan. 1st, 1907, to Oct. 28th, 1908, at 6% per annum.

6. Find the amount of \$1275 from July 4th, 1906, to Jan. 15th, 1908, at 8% per annum.

7. Bought a farm for \$5500 to be paid in 6 months, with interest at 4%; find the amount of the payment.

8. The interest on \$550 for 2 years is \$44; find the interest on \$100 for 1 year at the same rate.

9. The interest on \$840 for 1 year and 146 days is \$58.80; find the interest on \$650 for 2 years at the same rate.

10. What is the rate per cent., when the interest on \$1440 for  $1\frac{3}{4}$  years is \$72?

11. Find the rate per cent. per annum when \$400 amounts to \$448 in 3 years.

12. \$1200 amounts to \$1290 in 2 years and 6 months; find the rate per cent. per annum.

13. \$1500 amounts to \$1515 from Oct. 28th to Jan. 9th; find the rate per cent. per annum.

14. In how many years will \$500 amount to \$600 at 4%?

15. In what time will \$1260 amount to \$1340 at 3%?

16 In what time will \$100 amount to \$200 at 6%? What would be the amount of \$579.89 in the same time?

17. In what time will \$397.19 double itself at 4%?

18. In what time will any sum of money double itself at 3%?

19 At 4% for 3 years, the interest is what fraction of the sum loaned?

20. At 5% for 4 years, the interest is what fraction of the principal?

21. Calculate what fraction the interest is of the principal in the following:

(a) At  $7\frac{1}{2}\%$  for 4 years.

(b) At  $6\frac{3}{4}\%$  for 2 years.

(c) At  $3\frac{1}{2}\%$  for 6 years.

(d) At 5% for 20 years.

(e) At 8% for 6 months.

(f) At 6% for 4 months.

(g) At 10% for 292 days.

22. In one year the interest is  $\frac{1}{12}$  of the sum loaned; in how many years will the interest equal the sum loaned?

23. At 5% per annum, in how many years will the interest equal the sum loaned?

24. In what time will a sum of money double itself at  $6\frac{1}{4}\%$ ? At  $5\frac{1}{2}\%$ ? At 2%?

25. In what time will a sum of money treble itself at 4%? At 6%? At  $7\frac{1}{4}\%$ ?

26. The rent of a house, at \$20 a month, pays the taxes at  $1\frac{1}{4}\%$  on the value of the house, and  $6\frac{1}{4}\%$  on the money invested; what is the value of the house?

27. What monthly rent will pay the taxes at 1% of value, and 8% on the money invested in a house, for which \$8000 was paid?

28. What is the rate per cent. per annum when the interest on \$511000 for 5 days is \$280?

29. The interest on \$400 for 1 year at a certain rate, together with the interest on \$500 for the same time at double that rate, amounts to \$28; find the rates.

30. The interest on \$300 for 2 years at a certain rate, together with the interest on \$600 for 3 years at double that rate, is \$105; find the rates.

31. The interest on \$250 for 6 months at a certain rate, together with the interest on \$450 for  $1\frac{1}{2}$  years at  $\frac{3}{4}$  of that rate, is \$25.25; find the rates.

32. Find the amount of \$100 in 6 months at 4%.

33. What sum amounts to \$540.75 in 9 months at 4%?

34. Find the sum which in six months, at 5%, amounts to \$820.

35. What sum deposited in a bank now at 4% will, in 9 months, amount to \$1339?

36. What sum deposited at 6% will, in 219 days, amount to enough to pay the taxes on a building worth \$50000, taxed at 15 mills on the dollar?

37. A man has an offer of \$1500 cash, or \$1650 in 9 months; which is the better offer, money being worth 8% per annum?

38. \$250 amounts to \$275 in a certain time. What sum will amount to \$275 in one-half of the time?

39. A person borrowed money for 4 months at 8 per cent., and repaid principal and interest with \$462. How much was borrowed?

40. A person borrowed money for two years. For the first year he paid 5%, and the second year 6%. At the end of the time he paid back \$506.16. How much was borrowed?

41. A offers for a house \$2180, payable at the end of 3 years; B offers \$455 cash and \$455 at the end of each year for 3 years; C offers \$1600 cash. Which of these is the best offer, money being worth  $8\frac{1}{2}$  per cent?

42. If I borrow \$1200 for 3 years at 5%, with the understanding that the interest due at the end of each year shall form part of the principal for the next year, how much shall I have to pay at the end of the 3 years?

43. A man engaged in business was making 15% each year on his capital of \$15000; he gave up his business at a sacrifice of 10% of his capital, and loaned his money at 8%; what amount of income did he lose yearly?

44. What does a dealer gain by buying goods for \$2500 at 6 months' credit, and selling immediately for \$2800, banks paying 6% on deposits?

45. The interest on a sum of money amounts to  $\frac{5}{18}$  of the sum in  $7\frac{1}{2}$  years; find the rate per cent.

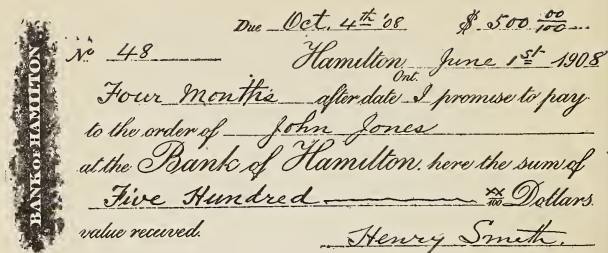
46. The interest on \$5000 in a certain time amounts to  $\frac{2}{3}$  the interest on \$100000 for 1 month at 6%; find the time.

47. In what time will \$1.33 $\frac{1}{3}$  amount to \$1.66 $\frac{2}{3}$ , at 5% per annum?

48. A dealer bought \$2000 worth of flour on 6 months' credit, and sold it immediately for 12 $\frac{1}{2}$ % advance. If from the proceeds he deposited in the bank sufficient money to amount to the \$2000 at the end of the six months, rate 5%, what sum had he left?

49. "To find the interest on a sum of money at 6%, multiply the sum by one-half of the number of months, and remove the decimal point two places to the left." Explain the rule.

## BANK DISCOUNT



The above is called a Promissory Note, or briefly a note.

The Face of the note is \$500.

It will be nominally due on Oct. 1st, 1908, but 3 additional days, called Days of Grace, are allowed by law in which it may be paid.

It is said to be legally due on Oct. 4th, 1908, and on that date John Jones should receive \$500 from Henry Smith.

If John Jones wishes to realize on the note before it is due, say on July 20th, he may discount the note at a bank.

In this case he will receive for the note its value at maturity, less a certain sum which is equivalent to the interest on \$500 for the actual number of days between July 20th and Oct. 4th, at whatever rate the bank is accustomed to charge.

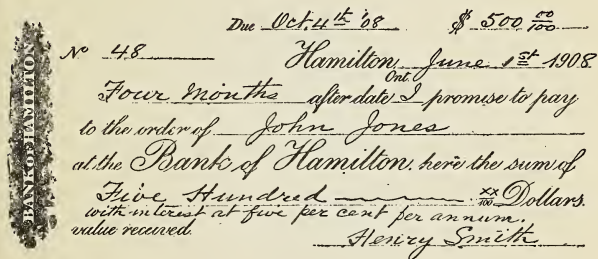
Thus, if the rate is 6% per annum, the interest on \$500 for 76 days (11 + 31 + 30 + 4) at 6% per annum will be  $\$500 \times \frac{6}{100} \times \frac{76}{365}$  or \$6.25.

John Jones will therefore receive \$500—\$6.25 or \$493.75.

This sum is called the proceeds of the note.

The \$6.25 is called the Bank Discount.

This note does not bear interest, that is Henry Smith will be required to pay only \$500 when the note is due.



Since interest is payable on this note at 5% per annum, the value of the note at maturity will be \$500, together with the interest on \$500 from June 1st to Oct. 4th at 5% per annum.

The interest on \$500 for 125 days at 5% = \$8.56,  
 $\therefore$  the value of the note at maturity = \$508.56,  
 $\therefore$  the discount =  $\$508.56 \times \frac{6}{100} \times \frac{76}{365}$  = \$6.35,  
 $\therefore$  the proceeds =  $\$508.56 - \$6.35$  = \$502.21.

The person who promises to pay a note is called the maker. The person to whom the money is to be paid is called the payee.

The day on which a note is legally due is called the day of maturity.

In Canada on all notes, except demand notes, the three days of grace are allowed. If the time is expressed in days, the actual number of days must be counted; if in months, calendar months are meant, not merely periods of thirty days each.

Thus, a note dated July 10th, payable three months after date, is nominally due on Oct. 10th, and legally due on Oct. 13th. If drawn on Oct. 31st, payable in four months, it is nominally due on the last day of February, and therefore the day of maturity would be Mar. 3rd.

If a note falls due on a Sunday or a legal holiday, it is payable on the following day.

A note made payable to the order of a person, or to the bearer, is **negotiable**, that is, it may be transferred from one person to another.

Notes, cheques and drafts are endorsed for the purpose of negotiating them, or for the purpose of additional security. **Endorsing** is signing the name across the back. An endorsement must be on the note itself, and must be an endorsement of the whole note and not merely of a part of it.

If a note is payable to two or more persons, who are not partners, all must endorse. Endorsement in **blank** is where the name only is written across the back of the note. It makes it negotiable by the bearer. A **specific** endorsement is one to a particular person as "Pay to A. Brown, or order." A. Brown must cash this himself, or endorse it before any other person can use it.

A **restrictive** endorsement is one which prohibits the further negotiation of the note, or merely gives authority to deal with it as thereby directed, as, "Pay A. Brown

only," or "Pay Bank of Hamilton, or order, for deposit to credit of John Jones," or "Pay Bank of Hamilton for collection on account of John Jones."

The endorser in effect guarantees to all subsequent holders: (1) that the instrument itself is genuine, (2) that he has a good title, (3) that he and all previous parties to it are competent to contract, (4) that the maker will pay it at maturity, (5) that in case the maker fails to meet it he will pay it himself.

If the note is properly presented and not paid by the maker at maturity, the endorser is entitled to have notice of the fact mailed to his given address if he is to remain liable. This is usually done by a notary in protesting it.

A note does not bear interest unless it is expressly stated in the note. If it bears interest, the three days of grace should be added in computing this interest.

If a note is not paid at maturity, interest can be collected from maturity to the date of payment, but only at the legal rate, which is at present five per cent per annum.

If a note bears interest, this interest at the rate specified can be collected until maturity, after which interest at the legal rate of five per cent. is allowed, unless the rate of interest is stated, in effect, as follows: "With interest at seven per cent. per annum until maturity and thereafter at the same rate until paid."

**Extract from Canadian Law.**—"Whenever any interest is by the terms of any contract, whether under seal or not, made payable at a rate per day, week, month, or for any period less than a year, no interest exceeding the rate of five per cent. per annum shall be recoverable,

unless the contract contains the express statement of the yearly rate of interest to which such other rate is equivalent."

Thus, on a note drawn with interest at one per cent. per month, only five per cent. per annum could be collected.

Simple interest only can be collected upon any contract unless compound interest is agreed upon.

### EXERCISE

Find (a) the *day of maturity*, (b) the *time* between day of discount and day of maturity, (c) the *discount*, (d) the *proceeds* of the following notes:

① \$1000 $\frac{0.0}{100}$ .

HAMILTON, *March 1st*, 1909.

Six months after date I promise to pay Oliver Bland, or order, the sum of One Thousand  $\frac{0.0}{100}$  Dollars, at my office here. Value received.

JOHN SMITH.

Discounted June 4th, 1909, at 5%.

2. \$1250 $\frac{0.0}{100}$ .

PARKDALE, *Jan. 15th*, 1909.

Three months after date I promise to pay to Dan. Wright, or order, the sum of One Thousand Two Hundred and Fifty  $\frac{0.0}{100}$  Dollars, at Standard Bank here. Value received.

SAMUEL NATTRASS.

Discounted Feb. 1st, 1909, at 6½%.

3. \$5640 $\frac{7.5}{100}$ .

BARRIE, Ont., *May 23rd*, 1908.

Four months from date I promise to pay James French, or order, at my office here, the sum of Five Thousand Six Hundred and Forty  $\frac{7.5}{100}$  Dollars. Value received.

ABRAM WILKES.

Discounted July 2nd, 1908, at 8%.

4. \$2769 $\frac{0.0}{100}$ .

GRIMSBY, Ont., *Dec. 1st*, 1908.

Ninety days from date I promise to pay William Barker, or order, the sum of Two Thousand Seven Hundred and Sixty-nine  $\frac{0.0}{100}$  Dollars. Value received.

ERNEST SMITH.

Discounted Dec. 24th, at 6%.

5. \$275 $\frac{0}{100}$ .HARRISTON, *April 1st*, 1909.

Four months after date I promise to pay Thomas Wright, or order, the sum of Two Hundred and Seventy-five  $\frac{0}{100}$  Dollars. Value received.

THOMAS JONES.

Discounted June 4th, 1909, at 8%.

6. \$4000 $\frac{0}{100}$ .TORONTO, *Nov. 29th*, 1908.

Three months after date I promise to pay George Holmes, or order, the sum of Four Thousand  $\frac{0}{100}$  Dollars at Bank of Commerce here. Value received.

SAM SMITH.

Discounted Dec. 1st, 1908, at 8%.

7. \$1234 $\frac{5}{100}$ .NEW YORK, *May 5th*, 1909.

Six months after date I promise to pay Henry Yorker, or order, the sum of One Thousand Two Hundred and Thirty-four  $\frac{5}{100}$  Dollars, at First National Bank here. Value received.

GEORGE GOULD.

Discounted June 4th, 1909, at 6%.

⑧. \$2356 $\frac{5}{100}$ .LONDON, *Feb. 14th*, 1909.

Sixty days after date I promise to pay to the order of Frank Smith, Two Thousand Three Hundred and Fifty-six  $\frac{5}{100}$  Dollars, at Imperial Bank here. Value received.

G. BROWN.

Discounted immediately at 6%.

⑨. \$400 $\frac{0}{100}$ .KINGSTON, *Jan. 29th*, 1909.

Thirty days after date I promise to pay William James, or order, the sum of Four Hundred  $\frac{0}{100}$  Dollars, at Bank of Toronto here. Value received.

HENRY SCOTT.

Discounted immediately at 8%.

10. \$576 $\frac{7}{100}$ .WHITBY, *Feb. 3rd*, 1908.

Four months after date we promise to pay to the order of Charles Beemer the sum of Five Hundred and Seventy-six  $\frac{7}{100}$  Dollars, at our office here. Value received.

A. WILMOT &amp; Co.

Discounted Mar. 1st, 1908, at 8%.

⑪. \$480 $\frac{0}{100}$ .PARIS, *Feb. 6th*, 1909.

Three months after date I promise to pay Samuel Cole, or order, the sum of Four Hundred and Eighty Dollars, at the Standard Bank here, with interest at 5%. Value received.

THOMAS JOHNSON.

Discounted Feb. 18th, 1909, at 6%.

(12) \$2000 $\frac{00}{100}$ .

OTTAWA, Mar. 4th, 1909.

Sixty days after date I promise to pay to the order of Henry Graham the sum of Two Thousand Dollars, at the Imperial Bank here, with interest at 6%. Value received.

ALEXANDER MCCUAIG.

Discounted immediately at 8%.

(13) \$4200 $\frac{00}{100}$ .

PORT HOPE, Aug. 25th, 1907.

Ninety days after date I promise to pay Thomas Scott, or order, the sum of Four Thousand Two Hundred Dollars, at the Bank of Montreal here, with interest at 7%. Value received.

GEORGE KELLY.

Discounted Sept. 1st, at 8%.

14. When must notes dated and drawn as follows be paid:

- (a) Jan. 30th, at 1 month?
- (b) Jan. 29th, at 1 month?
- (c) Jan. 28th, 1908, at 1 month?
- (d) Dec. 31st, at 4 months?
- (e) Nov. 29th, at 3 months?

15. The interest on any sum for 73 days, at 5%, is what fraction of the sum loaned?

16. The interest on a note for 95 days, at 6%, is what fraction of the face of the note?

17. A bank discounts a 92-day note at 6%. Find what fraction the discount is of the face value of the note.

18. A bank charges what fraction of the face of a note, when discounted 73 days before it matures, at 10%?

19. What fraction of the face value of a note does one receive from a bank which discounts it at 6%, 90 days before it is due?

20. A note is discounted 60 days before due at 8%, and the proceeds amount to \$360.20. Find the face value of the note.

21. What must be the face value of a note made June 1st at 3 months, and discounted immediately at 8%, to produce \$870?

22. What must be the face value of a note made May 25th at 4 months, and discounted June 3rd at 6%, to produce \$357.98?

23. A note for \$730 was discounted 45 days before it matured and produced \$724.60. What was the rate of discount?

24. A man received from a bank \$990 for a note of \$1000, 73 days before it was due; what was the rate of discount?

25. The discount on a note for \$1825, which matures on Aug. 1st, and was discounted on June 4th, was \$20.30; find the rate of discount.

26. The discount on a note of \$1460, discounted 40 days before it was legally due, was \$20; find the rate of discount.

27. For what sum must a note be drawn on June 1st, 1909, payable in 90 days, so that when it is discounted on June 14th, at 8%, the proceeds will amount to \$358.60?

28. A note for \$1460, discounted on May 23rd, 1908, at 6%, yielded \$1448.48. When was the note nominally due?

29. A ninety-day note, for \$292, was discounted on Dec. 20th, 1907, at 8%, and yielded \$289.12. On what date was the note drawn?

30. A sixty-day note, for \$1200, with interest at 6%, is discounted on the day it is made at 6%. Find the proceeds.

31. A bank discounts a note, legally due in 93 days, at 7% per annum. What rate of interest per annum does the bank receive on its money?

## PARTIAL PAYMENTS

\$3500 $\frac{0}{100}$

TORONTO, *March 1st*, 1908.

On demand, I promise to pay Henry Reid, or Order, the sum of Three Thousand Five Hundred Dollars, for value received, with interest at 5 per cent. per annum.

THOMAS KNOTT.

On this note the following payments were made:

May 13th, 1908, \$500.

Sept. 6th, 1908, \$1000.

1. What amount of interest was due on May 13th?

2. By how much did the payment on May 13th exceed the interest due?

3. By how much does the payment reduce the principal?

4. What amount of interest was due on Sept. 6th?

5. By how much did the payment on Sept. 6th exceed the interest due?

6. What amount did Knott owe Reid after making the payment on Sept. 6th?

7. How much was due Reid on Nov. 18th, 1908?

\$5000  $\frac{6}{100}$

TORONTO, *May 3rd*, 1907.

On demand, I promise to pay Thomas Scott, or Order, the sum of Five Thousand Dollars, for value received, with interest at 6% per annum. GEORGE COLE.

This note was endorsed as follows:

July 15th, 1907, \$40.

Sept. 28th, 1907, \$1200.

8. What amount of interest was due on July 15th?

9. By how much does the interest due on July 15th exceed the payment made on that date?

Note.—If the payment made at any time is less than the interest due at that time, this payment is added (without interest) to the next succeeding payment, and no reduction of principal is made until the sum of the payments exceeds the interest due at the time of the last payment.

10. What is the whole amount of interest due on Sept. 28th?

11. By how much does the amount of the two payments exceed the interest due?

12. How much does Cole owe Scott on Jan. 1st, 1908.

$\$400\frac{00}{100}$ MIMICO, *Jan. 1st, 1907.*

On demand, I promise to pay William Hill, or Order, the sum of Four Hundred Dollars, for value received, with interest at 6% per annum.

CALVIN KEMP.

On this note the following amounts were paid:

March 15th, 1907, \$20.

July 10th, 1907, \$6.

Sept. 20th, 1907, \$150.

13. How much was due on Dec. 24th, 1907?

14. The following payments were made on a demand note for \$1000, drawn March 1st, 1905, bearing interest at 8%: June 11th, 1905, \$300; Sept. 21st, 1905, \$10; Jan. 1st, 1906, \$100; June 10th, 1906, \$400. How much was due on June 1st, 1907?

15. A man bought a city lot for \$2000, giving \$500 cash, and making an agreement to pay 6% interest on the balance, with the privilege of paying off any part of the principal at any time when interest has been paid up to date. The transaction took place on April 1st, 1908. On Sept. 21st, 1908, he paid \$500; on Jan. 31st, 1909, he paid \$600. How much remained due on June 1st, 1909?

16. On a demand note of \$950, made Jan. 25th, 1908, bearing interest at 7% per annum, the following payments were made: March 2nd, 1908, \$225; May 5th, 1908, \$174.19; June 29th, 1908, \$187.50. What sum was due on Jan. 1st, 1909?

## EQUATION OF PAYMENTS

If a person loans me \$500 for 2 months, I might balance the favour by loaning him \$1000 for 1 month. For the use of \$500 for 2 months is equivalent to the use of \$1000 for 1 month.

Similarly we may say that the use of \$300 for 4 months is equivalent to the use of \$200 for 6 months, of \$100 for 12 months, or of \$1200 for 1 month.

If I owe \$400 due in 6 months and \$100 due in 11 months, when should I return the \$500 so as equitably to cancel the debt?

I am entitled to the use of \$400 for 6 months, which is equivalent to the use of \$2400 for 1 month, also to the use of \$100 for 11 months, which is equivalent to the use of \$1100 for 1 month.

Therefore I am entitled to the use of \$3500 for 1 month, which is equivalent to the use of \$500 for 7 months.

Therefore if I pay the \$500 at the end of 7 months it will equitably cancel the debt.

The meaning of this result is clearly that the sum of the interest on \$400 for 6 months and on \$100 for 11 months is equal to the interest on \$500 for 7 months.

The 7 months is here called the **Equated Time of Payment** of the whole debt.

If I wish to pay the whole debt at the end of 9 months, which is 2 months beyond the equated time, I should of course pay the \$500 together with the interest on it for the additional 2 months.

I buy merchandise as follows:

June 1st, \$400 on 30 days' credit,

June 10th, \$850 on 40 days' credit,

July 3rd, \$1200 on 30 days' credit.

On what day might the account be settled by the payment of the whole at one time?

On the first item I am entitled to the use of \$400 for 30 days after June 1st on the second to the use of \$850 for 40 days after June 10th, or 49 days after June 1st; on the third to the use of \$1200 for 30 days after July 3rd, or 62 days after June 1st.

The use of \$ 400 for 30 days = the use of \$12000 for 1 day

The use of \$ 850 for 49 days = the use of \$41650 for 1 day

The use of \$1200 for 62 days = the use of \$74400 for 1 day

\$2450

\$128050

The use of \$128050 for 1 day

= the use of \$2450 for  $\frac{128050}{2450}$  days or  $52\frac{13}{49}$  days.

∴ the payment of \$2450 would settle the account on the 53rd day after June 1st, or on July 24th.

### EXERCISE

1. Jones loans me \$200 for 4 months; for how many months should I loan him \$160 to balance the favour?

2. How many months' use of \$1000 is equal to the use of \$600 for 5 months?

3. Smith loaned me \$300 for 4 months, \$500 for 3 months, and \$450 for 2 months; how much money loaned Smith for 1 month would balance the favour?

4. Find the equated time of payment of the following: \$400 due in 10 days; \$800 due in 40 days; \$600 due in 60 days.

5. Find the average term of credit of \$500 due in 10 days, \$600 due in 12 days, and \$900 in 22 days.

6. I owe \$500, due 6 months ago; \$800, due  $1\frac{1}{2}$  months ago; how many months' interest should I pay on  $(\$800 + \$500)$ , in addition to paying \$1300, to cancel my indebtedness?

7. Find the average term of credit, and the equated time of payment from June 1st, of \$400 due in 30 days, \$600 due in 40 days, and \$500 due in 60 days.

8. Bought from Morton & Co., goods to be paid for on the following terms: \$1700, cash; \$1500 in 20 days; and \$1700 in 40 days; at what time might the \$4900 be paid in one payment?

9. A man owes a debt of \$2400, due in six months. He pays  $\frac{1}{3}$  of it in 3 months,  $\frac{1}{4}$  of it in 5 months; when does the remainder become due?

10. Bought from Eaton & Co., goods amounting to \$2400 on the following terms: \$400 cash, \$1200 due in 10 days, \$800 due in 30 days; find the equated time of payment.

11. A merchant bought goods from the wholesale house as follows: Mar. 4th, \$800 on 30 days' credit; Mar. 15th, \$1200 on 35 days' credit? When may the merchant equitably pay the \$2000?

12. A debt of \$5000 is due in 40 days; \$2000 is paid 15 days before the debt is due, and \$1500, 12 days before the debt is due; when should the balance be paid?

13. Bought from A. White & Co., on June 3rd, goods as follows: \$1800 cash, \$2400 on 30 days, \$800 on 60 days. I settled by paying \$4000 cash and giving my note for the balance. Find the time of the note.

14. I owe a friend \$400, due 40 days since; \$600, due now; \$1000, due in 30 days; find the equated time.

15. One-fifth of a debt was due 10 days ago; one-half is due now, and the balance in 20 days. Find the equated time of payment.

16 Find the equated time of the following sales:—

June 20th, a bill of \$500 at 30 days.

July 4th, “ “ 600 at 15 “

August 1st, “ “ 450 at 60 “

August 10th “ “ 800 at 90 “

17. Find when the balance of the following account should be paid:—

DR.				JOHN JONES.				CR.			
1908								1908			
May	1	To mdse. at 30 days....	\$800	May	20	By Cash.....	\$1000				
May	15	“ “ “ “ ....	600	June	15	“ “ ..... 500					
June	12	“ “ 60 “ ....	1000								

18 How much must be paid Jan. 1st, 1909, to balance this account allowing interest at 8% per annum?

## COMPOUND INTEREST

Banks which pay interest on deposits add the interest semi-annually or annually to the sum deposited.

Thus, if I deposit \$500 in a savings bank which pays interest annually at 4%, I would be entitled to \$20 interest at the end of the first year.

If I allow this interest to remain on deposit I would receive interest on \$520 during the second year.

The second year's interest would be \$20·80. My bank book would therefore show a balance to my credit, at the end of 2 years, of \$540·80.

In this example \$40.80 is called the **Compound Interest** on \$500 for 2 years at 4% per annum, compounded annually.

\$540.80 is called the **Amount**.

If the interest were compounded half-yearly, the interest for the first half-year would be 2% of \$500, or \$10,

$\therefore$  the amount on deposit at the end of the first half-year would be \$510.

The interest for the second half-year would be 2% of \$510, or \$10.20,

$\therefore$  the amount at the end of the second half-year would be \$520.20.

The interest for the third half-year would be \$10.40,

$\therefore$  the amount at the end of the third half-year would be \$530.60.

The interest for the fourth half-year would be \$10.61,

$\therefore$  the amount at the end of 2 years would be \$541.21,

$\therefore$  the total interest would be \$41.21.

Why is this interest greater than when the interest is compounded annually?

Find the amount of \$150 for 10 years at 5% per annum, compounded annually.

This problem might be solved in a manner similar to the preceding, but the result may be obtained much more readily as follows:

The interest for the first year =  $\frac{5}{100}$  of \$150,

$\therefore$  the amount =  $\frac{105}{100}$  of \$150 =  $\$150 \times 1.05$ .

The interest for the second year =  $\frac{5}{100}$  of  $(\$150 \times 1.05)$ ,

$\therefore$  the amount =  $\frac{105}{100}$  of  $(\$150 \times 1.05) = \$150 \times (1.05)^2$ .

Similarly,

the amount at the end of 3 years =  $\$150 \times (1.05)^3$ ,

and the amount at the end of 10 years

$$= \$150 \times (1.05)^{10}$$

$$= \$150 \times 1.62889 \text{ (see tables)}$$

$$= \$244.33,$$

$\therefore$  the interest for 10 years =  $\$244.33 - \$150$

$$= \$94.33.$$

The amount of \$1 for 5 years at  $3\frac{1}{2}\%$  per annum, compounded annually,

$$= \$ (1.035)^5 = \$1.19.$$

The amount of \$1200 for  $6\frac{1}{2}$  years at 4% per annum, compounded yearly,

$$= \$1200 (1.04)^6 \times 1.02$$

$$= \$1200 \times 1.26532 \times 1.02$$

$$= \$1548.75.$$

The amount of \$350 for 5 years at 6% per annum, compounded half-yearly,

$$= \$350 \times (1.03)^{10}$$

$$= \$470.37,$$

for the amount is evidently the same as it would be for 10 years at 3%, compounded yearly.

What principal loaned for 8 years at 4% per annum, compounded yearly, will amount to \$1000?

The amount of \$1 for 8 years at 4% =  $\$(1.04)^8$ .

Now  $\$(1.04)^8$  is the amount of \$1,

$$\begin{aligned}\therefore \$1000 \text{ is the amount of } & \frac{\$1000}{(1.04)^8} \\ & = \$1000 \times .73069 \text{ (see tables)} \\ & = \$730.69.\end{aligned}$$

### EXERCISE

**NOTE**—It is recommended that the first 17 examples in this exercise be solved without the use of the tables.

1. To what sum will \$100 amount if left on deposit in the Standard Bank for 3 years, interest calculated yearly at 4%?

2. What is the amount of \$1 in 3 years, at 4% per annum, interest calculated yearly?

3. What is the amount of \$100 in 4 years, at 5% per annum, interest calculated yearly?

4. What is the compound interest on \$150 in 4 years, at 5% per annum, interest calculated yearly?

5. Find the compound interest on \$875.25 in 3 years, at 4% per annum, interest calculated yearly.

6. Find the compound interest on \$1250 in 3 years, at 5% per annum, interest calculated yearly.

7. John Smith deposits \$100 in a Savings Bank at the beginning of each year, making the first deposit Jan. 1st, 1903. How much will there be to his credit Jan. 1st, 1909, the bank paying 4% per annum, calculated yearly?

8. What sum of money will give \$150 interest in 3 years, at 4% per annum, compounded yearly?

9. Find the amount accumulated at the end of 4 years by a man who invests \$150 now, and the same sum at the beginning of each succeeding year, at 4% compound interest, calculated yearly.

10 What is the difference between the simple and the compound interest on \$1275 for 3 years, at 5%, compounded yearly?

11 The difference between the simple and the compound interest on a certain sum for 4 years, at 6%, compounded yearly, is \$100. Find the sum

12 What sum of money loaned at 4% per annum, compounded yearly, will amount in 4 years to \$1200?

13. Find the amount of \$1200 in 2 years, at 6% per annum, interest calculated and added to the principal at the end of each half-year.

14. Find the compound interest on \$1450 in 1 year and 6 months, at 5% per annum, interest calculated half-yearly.

15 Find the compound interest on \$1 in 2 years and 73 days, at 5% per annum, calculated yearly.

16. What sum of money will yield \$400 interest in 2 years and 3 months, at 4% per annum, calculated yearly?

17 The rate of increase of the population of a town is 10% per annum the increase in the last 4 years is 13923. What is the present population?

18. Find the compound interest on \$1789.25, for 6 years, at 4% per annum, added yearly?

19. At 6% per annum, compounded yearly, find the amount of \$1 in  $8\frac{1}{2}$  years.

20. Find the amount of \$1 in 2 years and 9 months, at 6% per annum, payable yearly? Ans.  $\$(1.06)^2 \times (1.045)$ .

21. What sum, at 4% per annum, added yearly, will amount in  $2\frac{1}{2}$  years to \$16989.7728?

22. Find the sum which, in 2 years, at 4% per annum, payable half yearly, amounts to \$10824.3216.

23. The compound interest on a certain sum for 2 years and 73 days, at 5% per annum, compounded yearly, is \$82.82. Find the sum.

24. In how many years will a sum of money double itself at 10% per annum, compounded yearly?

25. At a certain rate, compounded yearly, the difference between the interest for the first year and that of the second is \$1, the difference between the interest of the second year and that of the third year is \$1.05. Find the rate per cent. per annum.

26. What rate per cent. per annum, compounded yearly, is equivalent to 3% per half-year, compounded half-yearly?

27. What rate per cent. per half-year, compounded half-yearly, is equivalent to 6% per annum, compounded yearly?

28. The compound interest on a sum of money, for 4 years, reckoned yearly, is  $\frac{34481}{160000}$  of the sum. Find the rate.

29. The difference between the interest at 10% per annum, added yearly, and that added half-yearly, for two years, is \$55.06 $\frac{1}{4}$ . Find the principal.

30. In what time will a sum of money double itself at 2 $\frac{1}{2}$ % compounded yearly? At 7% per annum compounded half-yearly?

31. If \$1 amounts to \$1.55297 in ten years, find the rate % per annum.

32. If \$1000 amounts to \$2406.62 in 18 years, find the rate % per annum.

33. In how many years will \$150 amount to \$301.83 at 6% per annum?

34. If \$100 amounts to \$120 in 4 years, to what sum would it amount in 8 years? In 2 years?

35. The time in which any sum of money will double itself at compound interest may be found approximately by dividing the rate % into 72. Examine how far the tables bear out the truth of this statement.

## PRESENT WORTH

A person owes me \$105 to be paid one year hence and he desires to pay it now. If I can loan my money at 5%, I should accept \$100 cash as the equivalent of the debt, for I could loan this \$100 at 5% and realize \$105 at the end of the year.

The \$100 is called the **Present Worth** or **Present Value** of \$105 due in one year.

The present worth of a sum, due at the end of any stated time, is therefore the principal which loaned for the given time would amount to that sum.

Thus, to find the present worth of \$1200 due in 9 months, when money is worth 6%, is the same as to find the principal which would amount to \$1200 if loaned for 9 months at 6% per annum.

The interest on \$100 for 9 mo. at 6% = \$4.50,

∴ \$104.50 due in 9 mo. is equivalent to \$100 now,

∴ \$1200 due in 9 mo. is equivalent to  $\frac{\$100 \times 1200}{104.50}$  now,  
= \$1148.32.

∴ the present worth is \$1148.32.

The difference between \$1200 and \$1148.32 or \$51.68 is sometimes called the **True Discount**. It is the interest on \$1148.32 for 9 mo. at 6%.

When the time is greater than one year, compound interest should be used.

Thus, the present value of \$1000 due in 6 years, money worth 5%,

$$= \frac{\$1000}{(1.05)^6} = \$1000 \times \frac{1}{(1.05)^6} = \$1000 \times .74622 \text{ (see tables)} \\ = \$746.22.$$

The present worth of \$400 due in  $5\frac{1}{2}$  years, money worth 6% per annum, compounded half-yearly,

$$= \frac{\$400}{(1.03)^{11}} = \$400 \times .72242 \text{ (see tables)} \\ = \$288.97.$$

Find the present value of \$100 a year, due at the end of each year for 4 years, if money is worth 5% per annum.

The present value

$$= \frac{\$100}{1.05} + \frac{\$100}{(1.05)^2} + \frac{\$100}{(1.05)^3} + \frac{\$100}{(1.05)^4} \\ = \$100 \left[ \frac{1}{1.05} + \frac{1}{(1.05)^2} + \frac{1}{(1.05)^3} + \frac{1}{(1.05)^4} \right] \\ = \$100 (.95238 + .90703 + .86384 + .82270) \\ = \$100 \times 3.54595 \\ = \$354.595.$$

Find the present value of a mortgage of \$1500 bearing interest at 6% per annum payable yearly, and having 3 years to run, money being worth 4% per annum.

On this mortgage there is due \$90 each year as interest, and \$1500 at the end of 3 years.

$\therefore$  the present value

$$= \frac{\$90}{1.04} + \frac{\$90}{(1.04)^2} + \frac{\$90}{(1.04)^3} + \frac{\$1500}{(1.04)^3} \\ = \$90 (.96154 + .92456 + .88900) + \$1500 \times .88900 \\ = \$249.759 + \$1333.50 \\ = \$1583.259.$$

## EXERCISE

1. Find the present worth of \$636, due in 9 months, money being worth 8% per annum.
2. Find the present worth of \$800, due in 8 months, money being worth 6% per annum.
3. Find the present value of \$1, due in 6 months, money being worth 8% per annum.
4. Find the present value of \$1, due in 2 years, money being worth 6% per annum, compounded yearly.
5. Find the present worth of \$8000, due in 2 years, money being worth 4% per annum, calculated yearly.
6. What sum should be deducted from \$1200, due in 6 months, if the debt is paid now, money being worth 4% per annum?
7. Find the true discount off \$1350, due in 9 months, money being worth 5% per annum.
8. Find the true discount off—
  - (a) \$485.50, due in 146 days, at  $7\frac{1}{2}\%$  per annum.
  - (b) \$1250.60, due in 1 year and 5 months, at 6% per annum.
  - (c) \$1234.56, due in 2 years, at 4% per annum, compounded yearly.
  - (d) \$17684.95, due in 3 years and 219 days, at 5% per annum, compounded yearly.
  - (e) \$1, due in 5 years, at 5% per annum, compounded yearly.
9. A farmer pays a yearly rental of \$400. What sum paid now would be equivalent to the next three years' rental, money being worth 5% per annum?
10. What sum, paid at the end of 2 years, is equivalent to \$400 paid at the end of each year, for three years, money being worth 4% per annum?

11. A owes B \$100, due in 2 years; \$150, due in 3 years; \$200, due in 4 years. What sum paid now would cancel the debt, money being worth 5% per annum?

12. The rent of a house is \$25 per quarter paid at the end of each quarter. What is the equivalent yearly rental paid in advance, money being worth 4% per annum?

13. The true discount off a sum for 3 months at 6% is \$7.75. Find the sum.

14. Which is the better, and how much, to buy a farm for \$3140 on 8 months' credit, or to accept a discount of 5% and pay cash, if money is worth 7%?

15. If money is worth 5%, what cash offer is equivalent to an offer of \$133.65 on 3 months' credit?

16. If goods which cost \$255 on 3 months' credit are sold for \$255 cash, what is the gain % if money is worth 8%?

17. Which is the better, and how much, to buy silk at \$4.18 per yard, payable in 9 months, or for \$3.95 cash, money being worth 6%?

18. A offers flour at \$7.50 a barrel on 8 months' credit, B offers the same grade of flour at \$7.25 a barrel on 3 months' credit. Which should I accept if money is worth 6%?

19. I buy goods for \$450 with a discount of 10% for cash. I pay cash for the goods and sell them immediately for \$433.50, payable in 3 months. If money is worth 6% find my gain %.

20. I buy goods for \$1545, payable in 6 months. I keep them in stock for 2 months and sell them for \$1624 on 3 months' credit. Find my gain if money is worth 6%.

21. Bought a farm for \$10000, payable one-half cash, the remainder in 1 year, with interest at 6%. I sell immediately for \$12000, payable in 3 months, with interest at 4%. What is my *present* gain, money being worth 5% per annum?

22. Find the difference between the bank and the true discount off \$530 due in 9 months, money worth 8%.

23. The difference between the bank and the true discount for 9 months at 4% is 90c. Find the sum.

24. If \$10 is the true discount off \$210 for 6 months, what is the true discount for 12 months at the same rate?

25. Find the present value of a mortgage of \$2000, having 4 years to run, bearing interest at 5% per annum payable yearly, if money is worth 6% per annum.

26. A mortgage for \$4000 has still  $2\frac{1}{2}$  years to run and bears interest at  $4\frac{1}{2}\%$  per annum, the first payment of interest being due in six months. How much is it worth now, if money is worth 5% per annum, compounded half-yearly?

27. How much may I borrow, so that I can repay it by paying \$1000 at the end of each year for 3 years, if money is worth 5% per annum?

## EXCHANGE

If a person owes a small sum of money to some one living in another place, he may send the money by registered letter, in which case the only expense is the amount of the postage.

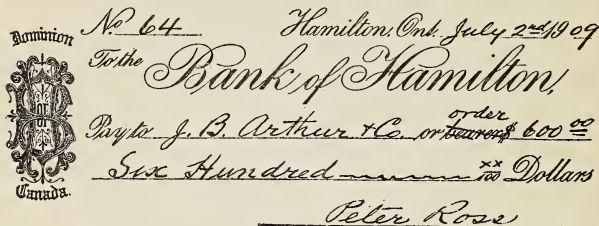
He may also pay it by means of a postal note, or by a money order purchased at the post office or from an express company.

In special cases the money might be paid by telegraph, at considerable expense, or the actual money might be paid to an express company for transmission.

In business, however, none of the above methods is usually followed.

If a Hamilton merchant (Peter Ross) buys goods from a Montreal firm (J. B. Arthur & Co.) to the amount of \$600, one way of making payment is for Peter Ross to send to J. B. Arthur & Co. his personal **Cheque** for \$600.

The cheque might be in the following form:



J. B. Arthur & Co., on receiving this cheque, would deposit it in the bank where they keep their account. This bank will send it to the Bank of Hamilton, which will deduct the \$600 from Peter Ross' account. The Montreal bank will probably make a small charge, which will be deducted from the amount credited to the account of J. B. Arthur & Co.

The amount charged is called the **Exchange**.

Another common method of making payment would be for Peter Ross to buy a **Bank Draft** from a Hamilton bank, directing a Montreal bank to pay J. B. Arthur & Co. the sum of \$600.

The draft might be in the following form:

<i>Bank of Hamilton</i>		<i>No. 3248</i>
<i>Hamilton, Ont.</i>		<i>July 2<sup>nd</sup> 1909</i>
Pay to the order of <i>Peter Ross</i>		<i>\$ 600<sup>00</sup></i>
<i>Six Hundred <sup>x</sup>/<sub>100</sub> ————— Dollars</i>		
To <i>The Royal Bank of Canada</i>	<i>C. B. Johnston</i>	<small>MANAGER</small>
<i>Montreal</i>	<i>A. P. Graydon</i>	<small>AGT.</small>

Peter Ross would endorse the draft thus:

Pay to the order of J. B. Arthur & Co.

PETER ROSS.

This draft might have been drawn payable to the order of J. B. Arthur & Co., in which case Peter Ross would not have been required to endorse it.

A draft is therefore the same as a cheque except that it is made by a bank and is drawn on another bank.

Banks usually charge a slight premium on the face of the draft.

Thus, the preceding draft for \$600 at 0.1% premium would cost  $\$600 + 0.1\%$  of \$600 or \$600.60.

Another method of effecting payment would be for J. B. Arthur & Co. to draw on Peter Ross for \$600. These drafts, or bills of exchange, are sometimes called Commercial Drafts and may be in the following form:

NO PROTEST  
FOR NON ACCEPTANCE ONLY

BANK OF MONTREAL

NO. 417

\$ 600<sup>00</sup>

At sight

DUE \_\_\_\_\_

MONTREAL July 2<sup>nd</sup> 1909

AFTER DATE PAY TO THE ORDER OF THE

BANK OF MONTREAL

Six Hundred <sup>00</sup>/<sub>100</sub> DOLLARS

VALUE RECEIVED AND CHARGE TO ACCOUNT OF

TO Peter Ross

Hamilton

Ont.

J. B. Arthur & Co.

R.D.&Co. 7991

J. B. Arthur & Co. would place this draft (called a **Sight Draft**) with the Bank of Montreal for collection. This bank would send it to some bank in Hamilton. The Hamilton bank would collect the \$600 from Peter Ross and would send it (or its equivalent) to the Bank of Montreal. The Bank of Montreal would then notify J. B. Arthur & Co. that it had been paid, and the amount, less the exchange, would be added to their account.

Thus, if the bank charged  $\frac{1}{4}$  of 1 per cent. exchange, the proceeds of the draft would be

$$\$600 - \frac{1}{4}\% \text{ of } \$600 \text{ or } \$598.50.$$

A draft payable **after sight** or **after date** matures as a promissory note.

A money order is always sold at a slight advance over its face value, and usually a bank draft costs more than its face value.

In each case the percentage of variation from the face value is called the **Rate of Exchange**.

If the draft costs more than its face value, exchange is said to be at a **Premium**.

If it costs its face value, exchange is at **Par**.

If it costs less than its face value, exchange is at a **Discount**.

The term **Domestic Exchange** is used to characterize transactions between parties in the same country and **Foreign Exchange**, in different countries.

Foreign drafts are called **Bills of Exchange**.

The **Par of Exchange** between Canada and any foreign country is expressed by stating the value of the monetary unit of that country in Canadian money. Thus, by referring to the table given, the par of exchange between Great Britain and Canada is £1 = \$4.86 $\frac{2}{3}$ , between France and Canada 1 franc = 19.3 cents, etc.

The rate of foreign exchange varies continually and is sometimes above par and sometimes below par.

The New York quotations for bills on London are always given in dollars per pound sterling.

Under the old par of exchange between England and America the dollar was valued at 4s. 6d. sterling; but by subsequent enactments the value of the dollar was reduced so that 4s. 6d. is now worth \$1.09 $\frac{1}{2}$ .

Therefore when sterling exchange is quoted at 1.09 $\frac{1}{2}$ , or briefly at 9 $\frac{1}{2}$ , it is at **par**, and any variation from this quotation is above or below par as the case may be.

$$\text{Since } 4\text{s. } 6\text{d.} = \pounds \frac{9}{40},$$

$\therefore$  under the old par of exchange

$$\pounds \frac{9}{40} = \$1,$$

$$\therefore \pounds 1 = \$\frac{40}{9} = \$4.44\frac{4}{9},$$

and under the new par of exchange

$$\pounds \frac{9}{40} = \$1.09\frac{1}{2},$$

$$\therefore \pounds 1 = \$\frac{40}{9} \times 1.09\frac{1}{2} = \$4.86\frac{2}{3}.$$

When sterling exchange is quoted at  $9\frac{1}{8}$ , the rate of exchange is

$$\begin{aligned} \pounds 1 &= \$4.44\frac{4}{9} \times \frac{109\frac{1}{8}}{100} \\ &= \$\frac{40}{9} \times \frac{873}{800} = \$4.85. \end{aligned}$$

Since the monetary unit in the United States is the same as in Canada, it should be noted, that business between a city in Canada and one in the United States is transacted, usually, as if they were both Canadian cities.

## VALUE OF FOREIGN UNITS IN CANADIAN MONEY

Country	Stand- ard	Monetary Unit	Value in Can. Money
Argentine Republic....	Gold..	Peso.....	\$0.965
Austria-Hungary .....	Gold..	Crown.....	.203
Belgium .....	Gold..	Franc .....	.193
Bolivia .....	Silver..	Boliviano .....	.478
Brazil .....	Gold..	Milreis .....	.546
Central America .....	Silver..	Peso .....	.393
Chile .....	Gold..	Peso .....	.365
China .....	Silver..	Tael (Canton).....	.642
Denmark .....	Gold..	Crown.....	.268
Ecuador .....	Gold..	Sucre .....	.487
Egypt.....	Gold..	Pound (100 piastres)	4.943
France .....	Gold..	Franc. ....	.193
German Empire .....	Gold..	Mark.....	.238
Great Britain. ....	Gold..	Pound Sterling.....	4.86 $\frac{2}{3}$
Greece .....	Gold..	Drachma .....	.193
Hayti.....	Gold..	Gourde.....	.965
India.....	Gold..	Rupee .....	.324
Italy.....	Gold..	Lira .....	.193
Japan.....	Gold..	Yen.....	.498
Mexico .....	Silver..	Dollar.....	.50
Netherlands.....	Gold..	Florin.....	.402
Norway .....	Gold..	Crown.....	.268
Panama .....	Gold..	Balbao .....	1.00
Persia.....	Silver..	Kran.....	.088
Peru .....	Gold..	Libra .....	.487
Portugal.....	Gold..	Milreis.....	1.08
Russia .....	Gold..	Rouble. ....	.515
Spain .....	Gold..	Peseta .....	.193
Sweden.....	Gold..	Crown .....	.268
Switzerland.....	Gold..	Franc .....	.193
Turkey .....	Gold..	Piastre.....	.044
United States .....	Gold..	Dollar.....	1.00
Uruguay.....	Gold..	Peso .....	1.034
Venezuela .....	Gold..	Bolivar.....	.193

## EXERCISE

1. A railway ticket from Liverpool to London costs 21s. 6d. What is its equivalent in our money?

2. Find the cost of the following sight drafts at the given rate of exchange:

(a) \$3560,  $\frac{1}{4}\%$  premium.

(b) \$7620,  $\frac{1}{8}\%$  premium.

(c) \$6410,  $\frac{1}{16}\%$  discount.

(d) \$2120,  $\frac{3}{8}\%$  discount.

3. What will a merchant pay for a draft for \$3750 at 40c. premium per \$1000?

4. What amount of draft on Montreal can be purchased for \$7500, at  $\frac{1}{4}\%$  premium?

5. A sight draft on New York for \$2700 was purchased for \$2673. What was the rate of exchange?

6. I draw on Brown & Co. for \$850. The bank charges  $\frac{1}{16}\%$  for collection. What are the net proceeds?

7. A telegraphic money order costs twice the rate for a ten-word message, together with 1% premium on the face of the order. A ten-word message between Toronto and Hamilton costs 25c. What will a telegraphic money order for \$465 cost?

8. Find, by using the table, the value in Canadian money of:

1. £100;      2. £20 10s.;      3. 60 francs;

4. 750 yen;      5. 220 lira;      6. 125 marks.

9. Find the difference in the value of 750 roubles, and 1450 crowns of Norway.

10. Find the cost of a draft on Berlin for 6720 marks when exchange is quoted at 94 $\frac{1}{2}$ c. for 4 marks?

11. What will it cost to cancel a debt in Yokohama for 2150 yen when exchange is at par?

12. The cost of a draft on Paris for 5742 francs was \$1100. What was the rate of exchange?

13. Find, in Canadian money, the cost of shipping 12000 bushels of wheat from Montreal to Liverpool at  $1\frac{7}{16}$ d. per bushel.

14. Find the cost of a bill of exchange on Paris for 1500 francs at 5·16 francs for \$1.

15. Find the cost of a 60 day bill of exchange on London for £1200, the rate of exchange being  $\text{£}1 = \$4\cdot80\frac{3}{4}$ .

16. Find the cost of a demand-bill on London for £1500, exchange being at  $9\frac{1}{8}$ .

17. What amount of demand-bill on Liverpool can be bought for \$2400, when exchange is at 8?

18. When \$7300 is paid in Toronto for a bill of exchange on Liverpool for £1500, how is exchange quoted?

19. A Canadian Company borrows in Paris 294,000 francs, for which it pays an annual interest of \$2920. This loan is transmitted through London when exchange on London is quoted at 25·30 francs, and sterling exchange,  $9\frac{3}{4}$ . Find the rate of interest the company pays on the money actually received.

20. A banker in Toronto remits \$10000 to Liverpool as follows: First to Paris, at 5·40 francs per \$1; thence to Hamburg, at 185 francs per 100 marks; thence to Amsterdam, at  $17\frac{1}{2}$  stivers per mark; thence to Liverpool, at 220 stivers per pound sterling; how much sterling money will he have in bank at Liverpool, and what will be his gain over direct exchange at 10% premium?

21. A commission agent sold goods to the amount of \$12500, charging a commission of  $2\frac{1}{2}\%$ ; with the net proceeds he bought a draft at  $\frac{3}{4}\%$  discount. Find the face of the draft.

22. A Toronto merchant owes 1800 francs in Paris; he buys a draft on London when sterling exchange is at 8; exchange between London and Paris 25·20 francs per £1. What does he pay for the draft?

23. The rate of exchange between London and New York is \$4·866564 for £1, and between London and Amsterdam 12·1071 florins for £1. Find the rate of exchange between New York and Amsterdam.

## General Problems

Each of the following sets of examples contains 100 problems, arranged in order of approximate difficulty.

Each exercise is intended to contain such a variety of examples, as will furnish a review of all the more important parts of the Arithmetic.

### EXERCISE I

1. A farmer bought 11 cows for \$253, and after keeping them 17 weeks, at a cost of \$1.75 a week each, he sold them for \$48 each. How much did he gain or lose by the transaction?

2. Find the interest on \$1022 for 93 days at  $5\frac{1}{4}\%$  per annum.

3. A bankrupt owes A \$4000, and B \$2700, and his assets are \$921.25. How should his assets be divided?

4. Three brothers buy a farm, the first taking  $\frac{2}{5}$  of it, the second  $\frac{1}{3}$ , and the third the remainder; the third paid \$1884. What did each of the others pay?

5. A boy bought apples at 5 for 6 cents, and sold them at 2 for 3 cents. What was his gain per cent?

6. Find the duty on 7200 pounds of sugar worth 6 cents a pound, the specific duty being  $\frac{1}{2}$  cent per pound, and the ad valorem duty  $25\%$ .

7. Find the cost of sowing a field 40 rods long and 30 wide, with oats worth 35 cents a bushel, if it requires 2 bushels of oats to sow one acre.

8. Find an agent's charges, who sells 3500 bushels of wheat, at 85 cents a bushel, on a commission of  $4\frac{1}{2}\%$ .

9. Find the difference between the simple and the compound interest on \$400 for 3 years at 7% per annum.

10. How much must a man pay for \$4000 stock at 87, brokerage  $\frac{1}{8}\%$ ?

11. I sold a horse for \$153, and lost 15% by the transaction. Find the cost price of the horse.

12. A wine merchant mixes 8 gallons of wine worth \$1.12 $\frac{1}{2}$  a gallon, 12 gallons worth \$1.25 a gallon, and 14 gallons worth \$1.50 a gallon, with 16 gallons of water, and sells the mixture at \$1 a gallon. Find his gain per cent.

13. Find the cost of fencing a 2 $\frac{1}{2}$  acre square field at 50 cents a rod.

14. Three men form a partnership, and invest their capital in the proportion of 3, 4, and 5. If they gain \$720, how should it be divided among them?

15. A merchant buys 4000 yards of carpet in England, at 4s. 6d. per yard. Find its value in Canadian currency, exchange being £1=\$4.87.

16. A mill valued at \$24000 is insured for  $\frac{3}{4}$  of its value at  $\frac{7}{8}\%$ . What is the amount of the premium?

17. Find my income from investing \$5100 in the 4 $\frac{1}{2}$  per cents at 85.

18. Find the area of a triangle whose sides are 19, 24, 36 ft. long respectively.

19. A grocer who throws off 5% for cash, sold the following: 3 lb. raisins, at 8 cents a pound; 4 $\frac{1}{2}$  lb. tea, at 50 cents a pound; 4 cans of salmon, at 25 cents a can; 8 lb. of sugar, at 6 $\frac{1}{2}$  cents a pound; 5 $\frac{1}{2}$  lb. of butter, at 20 cents a pound. Find his cash receipts.

20. At an election in a constituency in which the number of votes was 1800, the votes polled by the candidates were in the ratio of 7 to 5, and the majority for the successful candidate was 240. Find the number who did not vote.

21. Find the rate of simple interest at which \$137 will amount to \$152.07 in 2 years.

22. What is the duty, at 12 cents a pound, and 10% ad valorem, on 450 bags of wool, each containing 110 pounds, valued at 21 cents a pound?

23. Find the cost of carpeting a room 20 ft. long, 17 ft. 6 in. wide, with carpet 2 ft. wide, at \$1.50 a yard, the carpet running lengthwise, and without a pattern.

24. A merchant marked his goods at an advance of 20%, but afterwards sold them 40% less than this price. Required his loss per cent.

25. A bankrupt can pay 27 cents on the dollar. If his assets were \$480 more he could pay 30 cents. Find his liabilities and his assets.

26. A broker invests \$5176.50 in stock at 76. on  $\frac{1}{8}\%$  commission. What are his charges?

27. Divide \$4669 among 3 men, in the proportion of 5, 7, and 11,

28. A can do twice as much work in a day as B, but he works only  $\frac{3}{5}$  of the time. How should \$22, which they receive for their work, be divided?

29. A merchant at one time asked 25% less than cost for an article, but afterwards sold it for 25% more than this price. Find his loss per cent.

30. Find the net income of a man whose total income is \$900. on \$500 of which he pays 18 mills on the dollar taxation.

31. Find the cost of gravelling a walk, 1 yard in width, around the inside of a  $2\frac{1}{2}$  acre square field, at 15 cents a square yard.

32. In what time will \$212 amount to \$245.39, at 7% per annum, simple interest?

33. Simplify  $\frac{.0004 \times .00651 \times .03}{.0008 \times 3.1}$ .

34. A speculator invests some money in 3% stock at 72. What per cent. is he making on the money invested?

35. A drover shipped a car load of cattle to Toronto, and offered them for sale at an advance of 25% on the cost. The market being dull, he sold them for 14% less than his asking price, and gained \$170 on the load. Find the selling price of the cattle.

36. The expense of collecting taxes amounts to 2% of the total taxes. If a 19 mill rate gives the net taxes as \$36750, find the total amount of assessable property.

37. Divide \$201 between A and B, so that A may have \$12 more than half of what B gets.

38. If 2 pounds of tea are worth 3 pounds of coffee, and 14 pounds of coffee worth 8 pounds cocoa, and 2 pounds cocoa worth 24 pounds sugar, and 16 pounds sugar worth 11 pounds raisins, how many pounds of raisins would be worth 7 pounds of tea?

39. A merchant pays \$633.60 duty on an invoice of goods. If 20% of the goods be exempt from duty, and 22% is charged on the remainder, find the invoice price of the goods.

40. If a debt, after a reduction of 3%, becomes \$1008.80, what would it become after a reduction of 4%?

41. A commission merchant charges \$53.60 for buying goods on a commission of 2%. What sum must his employer remit to him to cover his charges and to buy the goods?

42. I mix 3 lb. of tea, worth 40 cents a lb., with 5 lb. of tea, worth 48 cents a lb. At what price per lb. should I sell the mixture to make a clear gain of 8%?

43. Three men form a partnership, contributing \$3200, \$4000, and \$4500, respectively. How should a gain of \$526.50 be divided?

44. Find the cost of painting a floor, 16 ft. in length and 12 ft. 3 in. in width, at 18 cents per square yard.

45. A man insures a house worth \$3200 for  $\frac{3}{4}$  of its value, at  $1\frac{1}{4}$ % premium. If the house be destroyed, find the total loss sustained by the company.

46. Find the compound interest on \$39 for 16 years, at 4% per annum.

47. A merchant's wholesale price is 15% advance on cost, and his retail price 10% advance on wholesale. Find his gain per cent. on cost at his retail price.

48. Find the difference between  $\frac{3}{4} \div \frac{1}{2}$  of  $\frac{5}{6}$ , and  $\frac{3}{4} \div \frac{1}{2} \times \frac{5}{6}$ .

49. The whole time occupied by a train, 90 yards long, and travelling at the rate of 18 miles an hour, in crossing a bridge is  $22\frac{1}{2}$  seconds. Find the length of the bridge.

50. Gunpowder is composed of nitre, charcoal, and sulphur, in the proportion of 33, 7, and 5. How many pounds of each in 60 pounds of gunpowder?

51. Find the volume of a right circular cone, whose height is 4 feet, and the circumference of the base 9 feet.

52. A speculator sold a lot gaining 20% of the cost. What per cent. of the proceeds did he gain?

53. Find the true discount off \$422.50 due in 9 months, at  $7\frac{1}{2}$ % per annum.

54. Reduce .714285 to a vulgar fraction in its lowest terms.

55. What principal will amount, at simple interest, to \$373.75 in  $2\frac{1}{2}$  years at 6% per annum?

56. Took a risk at  $1\frac{3}{4}$ %; reinsured  $\frac{2}{3}$  of it at  $2\frac{1}{4}$ %. My net premium was \$4.30. What was the amount of the risk?

57. A workman was hired for 40 days at 40 cents a day for every day he worked, but with this condition, that for every day he was **idle** he was to forfeit 16 cents; on the whole he made \$7.60. How many days did he work?

58. A man owes \$15000, bearing interest at 5% per annum; he pays at the end of each year, for interest and part payment of the principal, \$2500. Find the amount of his debt at the end of the third year.

59. If it costs \$2 a cord to saw wood into 3 lengths, what will it cost to cut a cord of wood into 4 lengths?

60. At a 14 mill rate a man who has \$400 of his salary exempt, pays \$7 taxes. What was his total salary?

61. The product of two numbers is 35643, and their quotient is 3. Find the numbers.

62. A bankrupt, whose liabilities are \$3000, can pay only 65 cents on the dollar, after the assignee receives \$50 for his services; what are the total assets?

63. A banker discounts a 70 day note at 8% per annum. What rate of interest is he making on his money?

64. If 275 lb. of sugar are bought at 6 cents a pound, and 10% of it be wasted, at what price, per lb., must the remainder be sold to make a gain of 14% on the whole transaction?

65. What is the difference between 40% discount, and 20, 10, and 10% discount?

66. How much carpet is wasted in carpeting a room, 20 ft. long and 11 ft. wide, with carpet (running lengthwise)  $\frac{3}{4}$  yd. wide, and having a complete pattern every 8 ft.?

67. Find the difference between the true and the bank discount off \$508, due in  $4\frac{1}{2}$  years, at 6% per annum simple interest.

68. A man in England invests a certain sum of money in Canada at 6%. All but \$400 of his income is taxed at 2%. If he pays \$18.28 taxes, find the amount of English money invested, exchange at par.

69. Milk is worth 28 cents a gallon, but by watering it the price is reduced to 5 cents a quart. Find the ratio of water to milk in the mixture.

70. How much will I save annually by investing \$7950 in the 3%'s, at 75, instead of in the 4%'s at 106?

71. What sum of money, with its semi-annual dividends of 5% invested with it, will amount to \$12750 in 2 years?

72. What is the value of property, if it be insured for  $\frac{3}{8}$  of its value, at a premium of 2%, and 5% of premium being charged for expenses, when the total cost of insuring is \$75.60?

73. What must I pay for a draft purchased at 2% discount to cancel a debt amounting to \$347.50?

74. I sold 2 books for 75 cents each, on the one I gained 20%, and on the other I lost 20%. How much did I gain or lose on the whole transaction?

75. Find the actual cost of goods which were purchased for \$510 on 3 months' credit, money being worth 8% per annum.

76. Reduce £350 5s. 7½d. to dollars and cents, a dollar being worth 4s. 2d.

77. A farmer sold oats for \$18.49, the price per bushel being as many cents as there were bushels. What was the price per bushel?

78. A bankrupt's assets are  $\frac{1}{3}$  of his liabilities, but  $\frac{1}{4}$  of the assets prove to be worth only 40 cents on the dollar. How many cents on the dollar can he pay?

79. What must be the face of a note made May 15th, at 5 months, and discounted June 27th, at 8%, to produce \$560?

80. A merchant sold 135 barrels of flour, part of it at \$5 a barrel, and the remainder at \$4 a barrel, and realized \$615. How many barrels of each kind did he sell?

81. A and B engage in trade; A puts into the business \$400 for 6 months, and B puts in \$300 for 7 months. How should a net gain of \$450 be divided?

82. A clerk enters upon work at a yearly salary of \$400, to be increased \$75 a year every 3 years. What difference would it have made in his total income for 10 years if he had received an annual increase of \$25 a year?

83. A factory is insured at 2%. If the premium, with \$1.50 for the policy, is \$362, and the insurance is on  $\frac{3}{8}$  of the value of the property, what is the value of the property?

84. What must be the least number of soldiers in a regiment to admit of it being drawn up 2, 3, 5, 6, 8, or 12 deep, and also of its being formed into a solid square?

85. I pay \$51 taxes on property worth \$4000, which is assessed at  $\frac{3}{4}$  of its value. Find the rate of taxation.

86. A manufacturer sold goods at 50, 25, 10% discount. What was the rate of discount?

87. Find the compound interest on \$48 for 8 years at 8% per annum, interest calculated half-yearly.

88. I bought tea at 55 cents a pound cash, and sold it immediately for 68 cents a pound on 3 months' credit. If money be worth 8%, find my immediate gain.

89. An importer paid 10% for freight and duty on goods invoiced at \$9875; he sold them at a clear gain of 20% to the retailer, who sells them at a gain of 25% to the consumer. Find the price paid by the consumer.

90. What rate of taxation must be levied on a corporation with \$2800000 assessable property to secure money to build a \$33950 school house, if the expenses for collecting amount to 3% of the total taxes?

91. The product of two numbers is 11154, their g.c.m. is 13. Find their l.c.m.

92. Which is the better, to buy flour at \$5 a barrel on 6 months' credit, or \$4.87 $\frac{1}{2}$  cash, money being worth 7%?

93. A man in New York purchased a draft for \$4680 on New Orleans, drawn at 30 days, paying \$4627.35. What was the rate of discount at which it was purchased?

94. A certain sum of money loaned at simple interest amounts to \$334.40 in 9 months, and in 7 months more to \$345.60. Find the sum and rate.

95. I owe \$719.92, and give my note for 60 days. What must be the face of the note to pay the exact debt, when discounted at 8% per annum?

96. Find the present worth of an annual payment of \$300 at the end of each year for 4 years, money being worth 6% per annum.

97. The ad valorem duty on 20% of the invoice price of a shipment of cigars was 40%, and on the remainder 35%; the whole duty being \$810, what was the invoice price?

98. A man invests \$7140 in stock at 84, and after it has advanced to 95 sells out. Find his gain.

99. A merchant sold a piece of cloth for \$24.60 and thereby lost 18%. What would have been his gain per cent. had he sold it for \$36?

100. Max Pierce rents a farm for \$300 a year, payable at the end of each year. If he does not pay the rent for 5 years, what will be the amount due, compound interest at 7%?

## EXERCISE II

1. A merchant mixes teas worth 35, 40, 60, 75 cents a lb., in equal quantities, and sells the mixture for 80 cents a lb. What percentage does he gain?

2. A man has \$10, \$5, and \$4 bills, the number of each denomination being proportional to 3, 4, and 6. If they amount to \$518 in all, how many has he of each denomination?

3. Fill in the blanks in the following bill of taxes:—

Amount of Assessment.....\$900

### *Taxes:*

Town rate, three and eight-tenths mills on the dollar.....

School Debenture By-Law, one and one-fourth mills.....

Harbour By-Law, two and six-tenths mills.....

Redemption of Debentures, one and four-tenths mills.....

School rate, five and one-tenth mills.....

High School rate, one and three-fourths mills.....

Total taxes.....

4. Find the sides of a rectangular field, containing  $3\frac{1}{4}$  acres, if the length is to the breadth as 3 to 2.

5. What is the least number, which, being a cube, is also divisible by 4, 5, 9, and 12?

6. Divide \$39 between A and B, so that 9% of A's share may equal 17% of B's.

7. The areas of the continents in square miles are as follows:—Europe, 3,780,000; S. America, 6,700,000; N. America, 8,750,000; Africa, 11,500,000; Asia, 16,500,000; if the number 3 represents the area of Europe, what whole numbers will most nearly express the areas of the others?

8. Goods to the amount of \$20000 were insured in 3 separate companies, for \$3000, \$4500 and \$6000, respectively. If the goods be damaged by fire, to the extent of \$9000, how much of the damage should each company sustain?

9. How much better is it to loan \$300 for 3 years at  $6\frac{1}{2}\%$  simple interest, than to loan it at 6% compound interest?

10. How many yards of paper, 30 inches wide, with a pattern every 18 inches, are required to paper the walls of a room 18 ft. long, 12 ft. wide, and 10 ft. high?

11. What is the difference between 60% discount, and 20% taken off 3 times?

12. A person invested \$2304 in the 3 per cents. at  $95\frac{1}{8}\%$ , and after receiving the half-year's dividend, sold out at  $94\frac{1}{2}\%$ . Find his gain or loss, brokerage being  $\frac{1}{8}\%$ .

13. Remitted \$4200, including commission, to my agent to purchase goods on a commission of 5%. What amount did he invest in goods?

14. A barrel of coal oil lost 20% by leakage, and the remainder was sold at a gain of 20%. Find the gain or loss per cent.

15. A note drawn on August 3rd for 4 months is discounted on September 12th at 9% per annum. What rate of interest is charged?

16. A lumberman sold 36840 feet of lumber at \$21.12 per M, and gained 28%. How much would he have gained had he sold it for \$17 per M?

17. The assessable property of a corporation is \$320000. What rate of taxation is levied, if the collector receives \$172.80 from a 3% commission?

18. A man loans \$375 at a certain rate of interest, and \$412 at a rate 2% higher. If his interest for one year from both investments is \$47.59, find the rate at which each was loaned.

19. The joint capital of A, B, C, and D is \$3150; A invests \$2 for every \$3 B invests, B \$4 for C's \$5, C \$6 for D's \$7. Required the amount invested by each.

20. A quantity of goods invoiced at \$1278, cost me in store \$1452.38, after paying the duty and \$14.63 for freight. What was the rate of duty?

21. A boy engages with a farmer for a year for \$40 and a suit of clothes. He leaves at the end of 9 months, and is entitled to \$25 and the suit. What was the cost of the suit?

22. The freight and mail earnings of a railroad company amount to \$476285.48, the passenger earnings to \$378567.29, the total expenses to \$564796.77, and the company was able to declare a dividend of 4%. How much stock had the company issued?

23. My salary is \$1200. If I pay 30% of it for board, 20% of the remainder for rent, 15% of the residue for clothes, \$71.20 for books, and loan 40% of the remainder, what per cent. of my salary is unexpended.

24. A grocer sells sugar at the rate of 49 ounces for 50 cents, which he bought at the rate of 50 ounces for 49 cents. Find his gain per cent.

25. A person gave 5 cents each to a number of beggars and had 14 cents left. He found that he would have required 22 cents more to enable him to give the beggars 8 cents each. How many beggars were there?

26. A and B can do a piece of work in 8 days, B and C in 10 days, and C and A in 12 days. Find when they will finish it, all working together.

27. What will it cost to gravel a walk, 2 yards wide, around the inside of a circular piece of ground whose diameter is 40 yards, at 14 cents a square yard?

28. I shipped my agent 6000 bushels of wheat, which he sold at 85 cents a bushel, and deducted 3% commission for selling it. He invested the net proceeds, less a commission of 2% calculated on the amount invested, in real estate. Find his total commission.

29. A man has \$400 of his income exempt from taxation, and on the balance he pays 2% income tax; if his net income is \$865.50, find his total income.

30. A man insures a house, worth \$4000, for  $\frac{3}{8}$  of its value, at 2% premium. If the house be destroyed, find the total loss sustained by the owner after one premium has been paid.

31. A broker receives \$25 for investing \$4300 in stock worth 86. What rate did the broker charge?

32. A grocer spent equal sums on tea, coffee and sugar; he sold them, making 12% on the tea, 8% on the coffee, and losing 15% on the sugar. His total gain being \$63.50, find the cost of each commodity.

33. The map of Ontario issued by the Crown Lands Department is drawn on a scale of 8 miles to an inch; on this map the Township of Scott measures  $1\frac{5}{16}$  inches in length and  $1\frac{1}{8}$  inches in width; how many acres does it contain?

34. A merchant lost 25% of certain perishable goods and sold the remainder at a gain of 30%. What was his gain or loss per cent?

35. A liquor dealer receives an invoice of 150 dozen of bottled porter, worth \$1.40 a dozen. If 10% of them are broken, what duty does he pay on the remainder, the rate being 24%?

36. A, B, and C enter into partnership, A contributes \$1200 for 4 months, B \$800 for 5 months, and C \$400 for 6 months. How should a gain of \$1960 be divided among them?

37. The simple interest on a sum of money for 3 years is \$45, and for  $5\frac{1}{2}$  years at 2% higher rate is \$110. Find the sum and rates.

38. Find in rods the length of the diagonal of a square field containing 40 acres.

39. In a 100 yard race, A can beat B by 5 yards and C by 10 yards. If B gives C 5 yards start in a 100 yard race, which wins and by how much?

40. A merchant sells 12 lb. of tea for what 17 lb. cost him. What advance per cent. on cost is he making?

41. A can do a piece of work in 5 days, B in 6, and C in 7. If they all work together at it and make \$21.40, how should the money be divided?

42. Find the compound interest on \$187 for  $4\frac{1}{2}$  years at 12% per annum, interest calculated every 4 months.

43. Three men form a partnership, each contributing the same amount of capital; one of them is appointed manager, and as such, is to receive  $12\frac{1}{2}\%$  of the total profits. When the profits are divided the manager receives altogether \$6000. What were the total profits?

44. A merchant paid \$36 for insuring 300 barrels of flour for  $\frac{2}{3}$  of its value at 3%. At what price, per barrel, must he sell it to gain 15% of the prime cost as well as the premium paid?

45. A man employed a number of men and 6 boys; he paid the men \$1.25 a day, and the boys 50 cents a day. The average price paid was \$1.15 a day. How many men were employed?

46. A bankrupt was able to pay 40 cents on the dollar, had not a debt of \$500 proved worthless; now he is able to pay only 24 cents on the dollar. Find the total amount of his liabilities.

47. Find the cost of fencing a farm in the form of a rectangle, whose sides are in the ratio of 4 to 5, and which contains 800 acres, at \$1.37 $\frac{1}{2}$  a rod.

48. Find the difference between the true and the bank discount off \$558, due 3 years hence, at 8% per annum simple interest.

49. A man sold two horses for \$200 each, on one he gained 25%, and on the other he lost an equal amount. Find the loss per cent in the latter case.

50. Sight exchange on Montreal for \$5000 cost \$5075. What was the rate of exchange?

51. A man insures property worth \$7840, so that in case of loss he may recover  $\frac{1}{2}$  of the value of the property and the premium paid for insuring, which was at 2%. What was the premium paid for insuring?

52. If I gain 30% of the proceeds in selling goods, what is my gain, per cent., on the cost?

53. If copper weighs 550 pounds, and tin 462 pounds to the cubic foot, what will be the weight of a cubic foot of a mixture of 6 parts copper to 5 parts tin?

54. Two-thirds of the selling price of certain goods is 10% less than cost. Find the gain per cent. at which the goods were sold.

55. A and B agree to pay their travelling expenses in the ratio of 2 to 3. A pays on the whole \$164, and B \$206. What has one to pay the other to settle the account?

56. Find the difference between the compound interest on \$200 for  $7\frac{1}{2}$  years at 7% per annum, interest calculated (1) half-yearly, (2) yearly.

57. The net amount received by a village for taxes is \$9690. The rate of taxation is 17 mills on the dollar, and the collector charges 5% of the total taxes. What is the amount of the assessment?

58. An agent remitted to his employer \$1872, as the proceeds of the sale of wheat, after deducting his commission of 4% for the transaction. What was the amount of the commission?

59. Which is the more profitable investment, 5% stock at 120, or 3% stock at 75?

60. A rectangular solid of metal has its length increased one-tenth, and its width one-seventh, by hammering; by how much has its thickness been diminished?

61. A sold \$2500 worth of goods to B, at a profit of 6%, and B sold them to C, at a loss of 5%. Find what C paid for the goods.

62. When exchange at New York on Paris is 5.16 francs per \$1. and at Paris on Hamburg 2.12½ francs per mark banco, what will be the price in New York of 26880 marks banco of Hamburg?

63. A can do as much work in 9 days as B can do in 10, but A works only 10 days for every 11 B works. How should \$59.70 be divided for work, under these conditions?

64. A merchant imported a quantity of goods, paying 25% of the invoice price for freight, duty, etc. He sold them for \$2925, losing 10% thereby. What was the invoice price?

65. A and B are partners. A's capital is to B's as 5 to 8; at the end of 4 months A withdraws  $\frac{1}{2}$  of his capital, and B  $\frac{2}{3}$  of his; at the end of the year their whole gain is \$4000. How much of the gain is each man entitled to?

66. Find the volume of a sphere whose surface measures 120 square inches.

67. A farmer sold 50 geese and turkeys for \$30.00. He received 50 cents each for the geese, and 75 cents each for the turkeys. Find the number of each sold.

68. How long will it take a man to walk around a square field whose area is 1.44 Ha., at the rate of 5 Km. per hour?

69. A jobber bought a bankrupt stock at 75 cents on the dollar and retailed it at 10% above the original wholesale price. His expenses were 6% of the total money received. Find his gain per cent. on the goods.

70. B owes a certain sum,  $\frac{1}{2}$  payable in three months,  $\frac{1}{4}$  in 5 months,  $\frac{1}{8}$  in 6 months, and the balance in 10 months. Required the average term of credit.

71. A commission merchant bought a lot, 40 feet frontage, with the commission he realized from selling wheat at 5¼%; the net proceeds of the wheat, after deducting the commission, being \$3790, find the price paid per foot for the lot.

72. A certain sum of money, at 6% interest, amounts to \$558.60 in a certain time. At  $8\frac{1}{2}\%$  for the same time it amounts to \$616.35. Find the time and sum (use simple interest).

73. A company took a risk at 4%, and reinsured  $\frac{3}{4}$  of it at 3%. The premium received exceeded the premium paid by \$27. Find the amount of the risk.

74. A merchant sold an article at a loss of 8%, but had he sold it for \$1.05 more, he would have gained 7%. What was the selling price?

75. A and B had a joint capital of \$1300, by which they gained \$715, of which A had for his share \$275 more than B. What did each contribute of the stock?

76. A can do a piece of work in 7 days, B in 10 days, and C in 14 days. How long will it take C to finish the work if A and B have both worked at it for 2 days?

77. Find the compound interest on \$32 for  $2\frac{1}{2}$  years at 12% per annum, interest payable every 2 months.

78. A city increases 13% annually in population for 3 years, and has at the end of that time 1,442,897 of a population. What had it at the beginning of the time?

79. A jeweller is required to supply 110 ounces of alloy, consisting of 3 parts of gold to one of silver. The only gold he has in stock is an alloy consisting of 22 parts of gold to 2 of silver. How much alloy must he use, and how much silver must he add to make up the order?

80. The difference between the true and the bank discount off a sum of money for 8 months at 9% per annum is \$4.95. Find the sum.

81. A man increases his income \$12, by transferring \$3000 stock from the 4 per cents to  $3\frac{1}{2}$  per cents at 75. Find the price of the former stock.

82. A man bought a farm for \$4000; at the end of 3 months he paid his taxes, levied on  $\frac{3}{5}$  of the purchase value at 18 mills on the dollar; in another 3 months he spent \$500 in improvements, and at the end of the year he sold for \$5500. Find his gain, money being worth 6% per annum.

83. Bought land at \$40 an acre. How much must I ask an acre that I may abate 25% from my asking price, and still make 30% on the purchase money?

84. A person had \$450; part of it he loaned at 5%, and the remainder at 7%, from which he received equal sums as interest. How much did he loan at 5%?

85. A speculator paid \$1400 for two lots, the price of one being 40% that of the other. He sold the cheaper lot at a gain of 50%, and the dearer one at a loss of 30%. Find his gain or loss on the whole transaction.

86. A certain article of consumption is subject to a duty of 6 cents a pound; in consequence of a reduction in the duty, the consumption increases one-half, but the revenue falls one-third. Find the duty per pound after reduction.

87. Find the weight of a sphere of tin of radius 3.5 dm., if 1 cu. cm. of tin weighs 7.3 grammes.

88. What will it cost to paint a cistern without a cover, inside and out, at 12 cents a square yard, if the cistern is 30 feet long, 21 feet wide, and  $8\frac{1}{2}$  feet deep?

89. Two men working together can perform a piece of work in 18 days. If it is worth \$126, and one of the men works 5 days less than the other, how should the money be divided?

90. A man having lost 20% of his capital, is worth exactly as much as another, who has just gained 12% on his capital; the second man's capital was originally \$4000. What was the original capital of the first man?

91. If 3% stock is selling at 84, what should be the price of 7% stock, to realize the same interest on the money invested?

92. A and B run a 100 yard race; A takes 8 steps while B takes 9, but 10 of A's are equal in length to 11 of B's. Who will win the race, and by how much?

93. A draft on Dublin for £360 costs \$1736. What was the rate of exchange?

94. An insurance company took a risk of \$4800 at  $2\frac{1}{2}\%$ , and immediately re-insured  $\frac{1}{3}$  of it in another company at  $3\%$ . If the property be destroyed, find the loss sustained by each company.

95. A certain number of men, and one-half as many women, were employed on a work; each man received \$1.25 and each woman 75 cents; their total wages being \$45.50, how many of each were employed?

96. A grain merchant sold 435 bushels of wheat at a profit of  $13\%$ , and 325 bushels at a profit of  $11\%$ , and realized \$1.10 more than he would have realized had he sold it all at a uniform profit of  $12\%$ . What was the price per bushel he paid for the wheat?

97. A farm is rented for \$700 a year, payable at the end of each year. What sum paid now is equivalent to 4 years' rental, money being worth  $5\%$ ?

98. What two numbers, each of four figures, have 101 for their greatest common measure, and 27573 for their least common multiple?

99. If silver is worth \$1.10 per ounce, and gold \$17 per ounce, find the weight of a ten-dollar coin containing 37 parts in 40 of gold, and the rest silver.

100. A note of \$2450, dated Halifax, June 1st, 1909, for 4 mo., bearing interest at  $6\%$ , is discounted at a bank on Aug. 15th at  $8\%$ . Find the proceeds.

### EXERCISE III

1. A man bought  $\frac{3}{8}$  of a vessel, and sold  $\frac{4}{5}$  of his share for \$11700 which was  $30\%$  above the cost. What was the cost of the vessel?

2. Find the cost of sowing a field, 40 rods long and 35 rods wide, with wheat at 70 cents a bushel, if it takes  $1\frac{1}{2}$  bushels to sow an acre,

3. The expense of constructing a bridge was \$873, which was defrayed by a tax on the property of the town. The rate of taxation was  $2\frac{1}{4}$  mills on the dollar, and the commission for collecting,  $3\%$ . What was the assessed value of the town?

4. What is the cost of 3825 pounds of sugar, bought at 4 cents a pound, on which is paid \$36.25 for freight, and  $2\frac{1}{2}$  cents a pound for duty after deducting 12% for waste?

5. Divide \$369 between A and B, so that A may have \$4.20 more than 14% of B's share.

6. A man has 11 hours at his disposal; how far may he ride in a coach, at 8 miles an hour, so as to return in time, walking back at the rate of 3 miles an hour?

7. A speculator buys stock when it is 25% below par, and sells it when it is 19% below par. What is his rate of gain?

8. A druggist gives a lb. Troy of certain goods instead of a lb. Avoirdupois. What is his gain and the customer's loss per cent?

9. A baker's outlay for material is 60% of the receipts, and his other expenses are 20% of the receipts. The price of flour falls 30%, and his expenses increase 10%. What should he now charge for a loaf, which formerly sold at  $7\frac{1}{2}$  cents, in order that he may make the same per cent. profit?

10. A and B can do a piece of work in 5 days, A could do it alone in 8 days. How long would it take B to finish the work if both together had worked at it for 3 days?

11. A merchant mixes 40 pounds of tea worth  $37\frac{1}{2}$  cents a pound, with 64 pounds worth 45 cents a pound; he sells 24 pounds of the mixture at 50 cents a pound. At what price per pound must he sell the remainder so as to clear 25% on his whole outlay?

12. My agent in Montreal sells a house and lot for me for \$8000, on a commission of  $1\frac{1}{4}\%$ , and remits to me the proceeds by a draft purchased at  $\frac{1}{4}\%$  premium. What sum do I receive from the sale of the property?

13. A person bought a house for \$3000. He insures it at  $1\frac{1}{4}\%$  for  $\frac{4}{5}$  of its value, pays \$1.25 a month water rates, and the assessor assesses it at  $\frac{5}{8}$  of its value (the rate of taxation being 19 mills on the dollar). If the property depreciates in value 5% during the year, what rate of interest does the owner make on his money, supposing that he receives \$35 a month rental for the property?

14. Find the difference in the perimeters of a square field, containing 4 acres, and a circular field, of the same size.

15. If 8% of the cost price of an article is equal to 6% of its selling price, what is the gain per cent?

16. Two men run around a circular track, 400 yards in circumference, at rates of 7 and 9 yards per second respectively. When and where will they first be together again, provided that they start together and run in the same direction?

17. A bankrupt has book debts equal in amount to his liabilities, but on \$24000 of them he realizes only  $66\frac{2}{3}$  cents on the dollar, and the expenses of the bankruptcy are 5% of the book debts. He pays 65 cents on the dollar. Find the amount of his liabilities.

18. The simple interest on a sum of money for a certain time at 6% per annum is \$62.10, and the interest on 3 times that sum for  $2\frac{1}{4}$  years at 4% per annum is also \$62.10. Find the first sum and the time.

19. What sum deposited in a bank at 4%, for 15 years, will amount to sufficient to produce thereafter an annual interest of \$450?

20. What shall I gain per cent. by purchasing goods on 9 months' credit, and selling them immediately for cash at the invoice price, money being worth 6%?

21. A farmer sold 16 bushels of corn and 20 bushels rye for \$30; 24 bushels of corn and 10 bushels of rye for \$27. How much per bushel did he receive for each?

22. I invest equal sums of money in the 4 per cents at 87, and in the 5 per cents at 102, and realize \$27 a year more income from the latter than from the former. How much do I invest in each?

23. A man's gross income is made up from \$1100 salary, and \$400 rent for a farm. His gross income is reduced to \$1187.50, on account of the following expenses:—\$1000 insurance at 2% on the house, 15 mills on the dollar taxation on salary and on  $\frac{2}{8}$  of the value of the farm, interest on a mortgage for  $\frac{3}{4}$  the value of the farm at 6%. Find the value of the farm.

24. A and B work 6 and 7 hours respectively for one day and receive the same wages; on the second day they work 7 and 8 hours respectively; they receive for their whole work \$18.10. How should it be divided?

25. Water expands 10% in freezing. Find the weight of water in a solid piece of ice 10 yards long, 5 feet wide, and 10 inches thick. (A cubic foot of water weighs 1000 oz.).

26. A grocer imported 120 cases of port wine, 36 bottles in each case. After 5% had been allowed for breakage, he paid an ad valorem duty of 20%; the freight and cartage expenses were \$80, and the whole cost was \$3773.60. What was the invoice price per bottle?

27. A merchant buys flannel at 32 cents a yard. At what profit per cent. must he sell it in order that the money he receives for 220 yards may be equal to his gain on \$480 outlay?

28. Find the present value of three sums of \$500 each, due at the end of 7, 8 and 9 years respectively, money being worth 5%.

29. A man pays \$342.50 for insuring goods. What is the total amount of insurance, if  $\frac{1}{6}$  of the amount is at a premium of 2%,  $\frac{1}{3}$  at  $2\frac{1}{2}\%$ , and the remainder at  $2\frac{1}{4}\%$ ?

30. Bought 600 barrels of flour at \$7 per barrel;  $\frac{1}{2}$  of which was to be paid in 6 months, and the balance in 9 months. If I paid cash in full on the day of purchase, find what I must pay, money being worth 6% per annum?

31. An agent sells 450 reapers for \$125 each. He is to be responsible for the bad debts, which amount to 10% of the entire sales, and is to receive 14% of the good debts for his commission. What are his net earnings?

32. Two men start together to go in the same direction around a circular track. The first goes 5 yards while the second goes  $4\frac{1}{4}$ . Where will they first be together?

33. A goes into business with \$1400 capital; after 3 months B joins him, putting in \$1600, and after 4 months more C also joins the firm, putting in \$1700. The gain at the end of the year was \$1191. How should it be divided?

34. The l.c.m. of two numbers is 924, the g.c.m. 12; one number being 84, find the other.

35. Find, to the nearest cent, the value of £460 10s. 8½d., exchange being at par.

36. Divide \$612 into 3 parts, such that the simple interest on the first for 2 years at 6%, equals the second for 3 years at 5%, equals the third for 4 years at 4%.

37. An article sold, at a loss of 25%, for \$5.62½. What would be the gain or loss per cent. if it sold for \$7.00?

38. Find the difference between the true and the standard time of the City of Hamilton, its longitude being 79° 54' west.

39. A man sold two lots, receiving  $\frac{3}{4}$  as much for one as for the other; on the cheaper one he lost 8%, and on the dearer one he gained 8%, but gained \$16 on the whole transaction. Find the proceeds of the two lots.

40. A grocer has a mixture of tea, weighing 80 pounds, part of it is worth 42 cents, and the remainder 37 cents a pound. If the mixture is worth 38½ cents a pound, find the number of pounds of each kind in the mixture.

41. A grocer sells his goods at 40% profit. He receives butter for his goods at a time when it is worth 25 cents a pound, but before he disposes of the butter 10% of it is worthless, and the remainder brings only 15 cents a pound. Find his gain or loss per cent.

42. A merchant marked his goods at an advance of 20% on cost, and in selling them he used a yard measure  $\frac{1}{4}$  inch too short; his entire gain being \$134.10, find the cost price of the goods.

43. A house worth \$4965 is insured for \$3000, so as to cover  $\frac{3}{5}$  of the value of the house, the premium of insurance, and charges amounting to 5% of the premium. What was the rate charged for insuring?

44. Three persons can together complete a piece of work in 45 days. It is found that the first does  $\frac{1}{3}$  of what the second does, and the second  $\frac{4}{5}$  of what the third does. In what time could each alone complete it?

45. Two brothers residing in different towns have gross salaries which differ by \$20; the one who has the larger salary has to pay an income tax of  $2\frac{1}{2}$  cents on the dollar, and as a consequence his net salary is \$1.40 less than his brother's, who pays an income tax of only 6 mills on the dollar. Find each of their salaries.

46. Find how much a merchant cheats a customer who buys goods to the amount of \$120, when he gives only 35 inches for a yard.

47. Find the difference between the true and the standard time of Goderich, its longitude being  $81^{\circ} 40'$  west.

48. A certain metal weighs 480 pounds per cubic foot, and is worth \$50.40 per ton. What will be the cost of a quantity of metal sufficient to make a mile of piping of 9 inch bore and  $\frac{3}{8}$  of an inch thick?

49. A sold goods which cost him \$10000 to B; B sold them to C, who sold them for \$11910.16. If they each gained the same rate per cent., find the rate of gain.

50. A grocer, by selling 5 lb. of tea for a certain price, gained 12%; afterwards he increased the price, giving only  $4\frac{1}{2}$  lb. for the same money. What per cent. did he make at the increased price?

51. Two men starting from the same point at the same time and travelling in opposite directions are 39 miles apart in 4 hours; but if they had gone in the same direction they would have been  $5\frac{1}{2}$  miles apart in 7 hours. Find their rates per hour.

52. A person owed \$2500 due in 8 months; he paid \$500 in 3 months, and \$800 in 6 months. When was the balance due?

53. A person sold a lot for \$600, gaining 14% of the proceeds. What would he have sold it for, had he gained 14% of the cost?

54. From a cask containing wine one-third is drawn off, and the cask filled with water; one-third of the mixture is then drawn off, and the cask is again filled with water. After this has been done 5 times altogether, what portion of the original quantity of wine will be left in the cask?

55. What is the amount at the end of 5 years, of \$100 invested at the beginning of each year, money worth 4%?

56. A merchant in Hamilton buys 25 tons of a cheap grade of coal for 75 cents a ton. He pays \$1.75 a ton for freight; it contains 25% of bituminous coal, on which he pays a specific duty of 60 cents a ton, and he also pays an ad valorem duty of 20% on the whole quantity, reckoned on the prime cost. Find his gain by selling it at \$3.50 a ton.

57. A builder pays 3 times as much for material as for labour; had he paid 10% more for material, and 6% less for labour, his contract would have cost him \$3637.92. What was his contract price?

58. What are the net proceeds from the sale of 1400 barrels of flour at \$7 per barrel, freight and storage being 25 cents a barrel, commission for selling  $2\frac{1}{4}\%$ , and for guaranteeing payment  $1\frac{1}{4}\%$ , calculated on sales?

59. A man increases his capital for 4 years at the rate of  $18\frac{3}{4}\%$  a year. At the end of the time his capital is \$1303.21. What was it at the beginning of the time?

60. A man insures a house, worth \$4900, so that in case of loss he can recover  $\frac{3}{5}$  of the value of the house and the premium of 2% paid for insuring. What was the amount of insurance?

61. Two trains, respectively 99 yards and 132 yards long, and moving on parallel tracks, pass each other in  $6\frac{3}{4}$  seconds when running in opposite directions. When moving in the same direction the one passes the other in  $47\frac{1}{4}$  seconds. Find their rates per hour.

62. Bought 3000 bushels of wheat at \$1.15 per bushel, payable at the end of a year. I immediately sell it at \$1.04 per bushel, cash, and put the money at interest at 10%. How much will I gain or lose by the transaction at the end of the year?

63. A person invested \$8001 in the 4 per cents at 95 $\frac{1}{4}$ , and when they rise to 98 sells out and invests in the 3 per cents at 84. What amount of the latter stock does he obtain?

64. The stocks of 3 partners, A, B, and C, are \$4000, \$6500, and \$3500, respectively; their gains are \$960, \$2340, and \$1400, respectively. If B's stock is in trade 3 months longer than A's, what time was each stock in trade?

65. A commission merchant had shipped to him 1000 barrels of flour, and 4000 bushels of wheat. He paid  $12\frac{1}{2}$  cents a barrel for the storage of the flour, 3 cents a bushel for the wheat, \$67.48 freight and duty. If he sold the flour at \$8 a barrel, on a commission of  $1\frac{1}{2}\%$ , and the wheat at 90 cents a bushel, on a commission of 2 cents a bushel, what sum did he remit to his employer?

66. A farmer borrows \$1500 at 7%, and agrees to pay both principal and interest in 2 equal annual payments. What is the amount of each payment?

67. A druggist marks his soda at 30% profit, but by a mistake sells a pound by Troy weight. What percentage does he really gain on the pound?

68. Two circular plates of solid gold, each one inch thick, the diameters of which are 9 and 12 inches respectively, are melted into a single circular plate one-half inch thick. Find its diameter.

69. The assets of a bankrupt, which amount to \$7290, are to be divided between two creditors, A and B, whose interests in the business are as 4 to 5. A acts as assignee, and for his work receives  $3\frac{1}{2}\%$  of the assets. The remainder is to be divided between them in the above ratio. What share of the assets does each man receive?

70. A grocer has 280 lb. of tea, of which he sells 80 lb. at 30 cents a pound, and gains only 8% at this price; he now raises the price so as to gain 10% on the whole outlay. What does he now sell at a pound?

71. A, B, and C work 6, 7, and 8 hours a day, respectively, for 2 days, and receive the same daily wages. They finish the work in 3 days more by each working one hour a day extra. If \$91.30 is paid for the work, what should each man receive?

72. A merchant sent his agent \$6150, with instructions to deduct his commission at  $2\frac{1}{2}\%$ , and invest the remainder in flour at \$6 a barrel. If the cost of freight and insurance amounts to \$250, at what must the flour be sold a barrel so as to make a clear profit of 15%?

73. How much stock must be sold, at 72, to pay a debt of \$7470, 9 months before it becomes due, true discount being allowed, at 5% per annum?

74. A merchant sold  $\frac{5}{9}$  of a lot of goods at a gain of 25%, and the remainder for 85% of cost; his whole profit was \$260. Had he sold  $\frac{5}{9}$  at a gain of 15%, and the remainder for  $\frac{3}{4}$  of cost, what would have been his gain or loss?

75. How much tea, worth 54 cents a pound, must I mix with 34 pounds, worth 60 cents a pound, so as to form a mixture worth  $57\frac{2}{3}$  cents a pound?

76. J. Brown and L. Stuart enter into partnership, and agree that Brown is to receive \$400 of the profits for managing the business, and the balance of the profits is to be divided in proportion to their investments; Brown puts into the business \$1300 and Stuart \$1500. If Brown received \$350 more of the profits than Stuart did, what were the total profits?

77. A person invested in 3% stock, so as to receive  $5\frac{1}{4}\%$  clear on his investment, after paying an income tax of 20 mills on the dollar? What was the market price of the stock, brokerage being  $\frac{1}{2}\%$ ?

78. A merchant ships \$2700 worth of goods to his agent to sell. His agent's terms are  $7\frac{1}{2}\%$  commission and guaranteed payment of sales, or 4% without the guarantee. The merchant accepts the latter terms, but part of the sales being worthless, he loses \$21.50 by not accepting the other plan. How much did he lose on bad debts?

79. A man buys 75 acres of land at \$140 an acre, and sells a portion of it at an advance of 50% on the cost. He realized on the part sold a sum equal to the cost of the entire lot. How many acres did he sell?

80. A merchant wishes to transmit 5600 marks banco to Hamburg. He finds exchange between Montreal and Hamburg to be 36 cents for 1 mark. The exchange between Montreal and London (Eng.) is \$4.83 for £1; that between London and Paris is 26 francs for £1; and that between Paris and Hamburg is 47 francs for 25 marks. By which way should the merchant transmit?

81. A father willed his property, amounting to \$26600, to his two sons, aged 14 and 17 years, respectively, in such a way that the sums loaned at 6% per annum, simple interest, would be equal when they became of age. How did he divide it?

82. A man rows 3 miles down a stream in 30 minutes, and back in 45 minutes. Find the rate of the stream, and of the man in still water.

83. Three men rent a pasture for \$165. The first puts in 4 horses for 8 weeks; the second 5 horses for 11 weeks, and the third 9 horses for 5 weeks. How much ought each to pay?

84. Took a risk at  $1\frac{1}{4}\%$ ; reinsured \$4000 of it at  $2\%$ ; and \$6000 more at  $2\frac{1}{4}\%$ . My net premium was \$26.50. What amount was insured?

85. A merchant buys sugar at 5 cents a pound; in selling it he wastes  $7\%$ , and  $20\%$  of the sales are bad debts. At what price per pound must he sell the remainder so as to gain  $24\%$  on the whole transaction?

86. If \$5 be allowed as discount off a bill of \$125 due a certain time hence, what would be the discount allowed if the bill had twice as long to run?

87. A square box, whose depth is 14 inches, has a capacity of 4 cubic feet 113 cubic inches. Find the length of a side.

88. A, B, and C do a piece of work, and are paid \$42 for it. The money is divided according to the efficiency and time each worked; A's efficiency is to B's as 2 to 3, B's to C's as 4 to 5; A worked 6 days, B 7 days, and C 8 days. How should the money be divided?

89. A merchant sold 276 yards of cloth at a profit of  $15\%$ , and 398 yards at a profit of  $7\%$ , and found that had he sold it at a uniform profit of  $11\%$ , he would have realized \$4.88 more than he did. What was the cost price of the cloth?

90. A merchant sold cloth at  $20\%$  gain, but had it cost \$49 more, he would have lost  $15\%$  by selling at the same price. What did the goods cost?

91. A house that cost \$15500 rents for \$155 a month. It is insured for \$10850 at  $\frac{4}{5}\%$  yearly; the taxes are 15 mills on an assessment of \$12450, and \$346.45 is spent each year on repairs. What rate of interest does the investment pay?

92. A contractor sends in a tender of \$11000 for a certain work, with nothing to be advanced till the work is completed; a second sends in a tender for \$10000, but stipulates to be paid \$2000 every 4 months. Find the difference between the tenders, supposing that the work cannot be completed before the end of 2 years, and money to be worth 2% per term of 4 months.

93. A man increases his annual income \$72 by transferring \$21000 stock from the 3%<sup>s</sup>, at 78, to the  $4\frac{1}{2}\%$ 's. Find the price paid for the latter stock.

94. The compound interest on \$250 for 2 years is \$50. Find the rate.

95. A merchant buys goods for \$304.50 on a credit of 3 months. At what price must he sell them, on a credit of 8 months, to make a ready gain of 25%, money being worth 6%?

96. A man sold an article so as to gain 25%. If he had sold it for \$150 less, he would have lost 5%. Find the cost of the article.

97. A man sold two horses for equal amounts, gaining 10% on one, and losing 10% on the other. If he lost \$4 on the whole transaction, find the cost of each horse.

98. An agent sold a consignment of wheat for \$4120, and invested this sum, less his commission, in corn. His total commission on both transactions amounted to \$240. What rate did he charge, the rate in each case being the same?

99. A person has a sum of money to invest which will buy \$1200 more 4% than  $4\frac{1}{2}\%$  stock, when the former is selling at 87 and the latter at 99. Find the difference in the incomes from investing the sum in these securities.

100. J. Brown owes \$400 due in 6 months, \$320 in 9 months, and \$280 in 12 months. Required the average term of credit.

## EXERCISE IV

1. A merchant buys 1440 yards of cloth. He sells  $\frac{1}{3}$  of it at a gain of 8%,  $\frac{1}{3}$  at a gain of 12%,  $\frac{1}{3}$  at a gain of 14% and the remainder at 9% loss. Had he sold the whole at a gain of 5% he would have received \$23.50 more than he did. What was the cost price per yard?

2. On March 23rd a bank gave me \$845 for a note of \$860, which it discounted at 8%. When was the note due?

3. Does a merchant gain or lose, and how much per cent., by selling half of a barrel of sugar, giving only 15 ounces to the pound, and the other half giving 17 ounces to the pound?

4. How many bricks, 9 inches long,  $4\frac{1}{2}$  inches broad and 4 inches thick, will be required to build a wall 45 ft. long, 17 ft. high and 4 ft. thick, supposing the mortar to increase the volume of each brick  $6\frac{1}{4}$  per cent.?

5. A and B had the same amount of money, A lost 39% of his in speculation, and B gained 45% of his in another speculation; both together now have \$164.80. What sum had each at first?

6. In a poorhouse, where there is always the same number of inmates to be fed, the contract price of meat rises 25%, and the daily allowances to each person, at the same time, is reduced from 9 ounces to 7 ounces. If the yearly charges for meat are now \$714, what were they before the rise in price?

7. A man insures his house so that in case of loss he may recover the value of the house and the premium of insurance at  $1\frac{1}{4}\%$ . The house is destroyed by fire and  $\frac{3}{5}$  of the claim is allowed; he finds that he receives \$1550 less than the value of the house. Find the value of the house and the premium.

8. Find the present worth of a promissory note of \$728, due 2 years hence (a) at simple interest, (b) at compound interest, money being worth 6 per cent.

9. A circular race-course is 22 yd. wide and has an area of 12 acres. Find the diameter of the inner circle.

10. If stock at 7% discount will pay 6% interest on the investment, at what rate of discount would it have to be bought to pay 9% interest?

11. A merchant paid \$268.80 duty on 24 casks of wine worth \$1.75 a gallon, a leakage of  $11\frac{1}{9}\%$  being allowed at the custom house. What was the invoiced number of gallons in each cask, the rate of duty being 20%?

12. On June 5th A bought goods to the value of \$5191.53, and gave in payment his note due in 3 months. What must be the face of the note so that when discounted at 7 per cent. it will realize the amount required?

13. A consignment of 1000 bbl. of flour was sent to a commission merchant, with instructions to sell it and remit the net proceeds by draft. The consignee pays for freight and other expenses \$240.80, sells the flour at \$8.50 per barrel, charges  $2\frac{1}{2}\%$  commission, and pays  $\frac{3}{4}\%$  premium for the draft; how much does the consignor receive?

14. A merchant bought 100 barrels of flour, part at \$7 a barrel, and the remainder at \$5 a barrel; by selling the former at 15% gain, and the latter at 14% loss, he just cleared himself on the transaction. How many barrels of each did he buy?

15. A speculator bought 368 acres of land at \$57.50 an acre, borrowing the money at 4%. At the end of the year he sells  $\frac{3}{4}$  of it at \$63 an acre, and the remainder at \$50 an acre. How much does he lose by the transaction?

16. A and B form a partnership, A contributing \$2700, and B \$3700; they agree that 20% of the total profits, shall be placed to the credit of the firm annually, and the remaining profits be divided in proportion to the capital invested by each. At the end of the year A's share of the profits was \$675. Find the percentage of profit realized on the entire capital.

17. Find how much a person will save in a year if, instead of renting a house at \$30 a month, he borrows \$3000 at 6% and buys one, which he insures for its full value at  $1\frac{1}{4}\%$  annually, and which is assessed at its full value (the rate of taxation being 19 mills on the dollar). He also pays \$1.25 a month water rates, and in selling the property at the end of the year receives 2% less for it than he paid.

18. A man owes \$700, due in 6 months; he pays \$300 down. What extension of time ought to be allowed for the payment of the remainder?

19. A commission merchant sold flour on a commission of  $\frac{1}{2}\%$ ; with the net proceeds bought tea on a commission of  $\frac{1}{3}\%$ ; the total commission was \$50. Find the cost of the tea.

20. A lot of land in the form of a rectangle contains 6 ac. 132 sq. rods; its length is to its width as 21 to 13. Find the number of rods of fence required to inclose it.

21. In what time will \$30441 gain \$2210·10 if, at the same rate, the gain on \$24944·10 for 1 year and 15 days is \$2596·92? What is the rate per cent. per annum (365 days to a year)?

22. A builder takes a contract, and at the time the material will cost just twice as much as the labour. When one-half of the material is secured, and one-third of the work performed, the cost of material rises 5%, and at the same time the cost of labour falls 8%. As a consequence, the builder saves \$10 on his first estimate. What was the first estimate?

23. I bought French goods for 7490 francs, and paid an import ad valorem duty of 15 per cent. I sold the goods for £420. Find my gain or loss in dollars and cents, if £1=25·22 fr.= \$4·87.

24. A man borrows \$2500, and agrees to pay both principal and interest in 3 equal annual payments. What is the amount of each payment, interest being at 5%?

25. What is the average time at which the following bills become due: Feb. 12, 1909, \$300 on 3 months' credit; March 6, 1909, \$275 on 30 days' credit; May 17, 1909, \$412 on 6 months' credit; Aug. 7, 1909, \$500 on 2 months' credit?

26. A tradesman marks his goods at two prices, one for ready money and the other at a credit of six months. What is the ratio of these prices, if money is worth 10 per cent.?

27. A merchant, after reducing the marked price of an article by three successive equal rates of discount, sold for \$21·87; the marked price being \$30, what was the rate of discount?

28. The duty on wine containing 26% or less of spirits is 25 cents per gallon, and 30% ad valorem; for every degree above 26%, 3 cents more per gallon. A man wishes to import 100 gallons of wine 30% strong, bought at \$2 per gallon; he orders enough water to be mixed with it to reduce the strength to 26%. Does he gain or lose in the amount of duty paid, the total value of the wine being reduced 10% by the mixing?

Does he gain or lose on the whole?

29. A person pays \$181.50 for £37 10s. What per cent. premium is sterling exchange?

30. A man removed from a town where his income tax was 16 mills on the dollar, to another where the tax was only 7 mills on the dollar, and although in the latter place his salary was \$500 more than in the former, his taxes were \$6.40 less. Find his income tax in the latter place.

31. \$10.50 is the compound interest on a certain sum for the second year, \$11.025 is that for the third year. Find the rate and the sum.

32. A certain railway company is able to declare a dividend of  $5\frac{1}{2}\%$  on its entire stock, but there being \$150000 of preference stock, which is guaranteed 8%, the ordinary stockholders receive only 4%. Find the total stock of the company.

33. Bought 64 yards of cloth at \$5.70 a yard. If it shrinks 5% in length, find the selling price per yard to gain 20%.

34. A grocer sells coffee for cash at a gain of  $33\frac{1}{3}\%$  on cost. He also sells on credit, giving 8 pounds for what would buy 9 if paid in cash. How much per cent. above cost is his credit price?

35. A merchant bought a quantity of tweed and marked it at an advance of 25% on cost, and in selling it he used a yard measure  $\frac{3}{4}$  inch too short, his entire gain being \$124.80. Find the cost price of the cloth, and the amount the merchant gained by his dishonesty

36. A man is owed \$2500, of which his lawyer collects 75% and charges 4% of the receipts for collecting. On suing for the remainder he gets 40% of it, of which his lawyer's fees are 10%. What per cent. of the debt does the creditor obtain?

37. If the interest on one dollar for one year at one per cent. per annum be the unit, what number will represent what a person owes for the use of \$325 for  $6\frac{1}{2}$  years at 6 per cent. per annum?

38. When exchange at New York on Paris is 5-16 francs per \$1, and at Paris on Hamburg  $2-12\frac{1}{2}$  francs per mark banco, what will be the price in New York of 11520 marks banco of Hamburg?

39. A commission merchant receives 125 bbl. of flour from A, 150 bbl. from B, 225 bbl. from C; he finds on inspection that A's is 10 per cent. better than B's, and C's is  $5\frac{5}{11}$  per cent. better than A's. He sells the whole lot at \$7.00 per barrel, charging 4 per cent. commission. What sum must he remit to each?

40. What is the cost of polishing a cylindrical marble pillar, 2 ft. 6 in. in diameter and 12 ft. long, at \$1.25 a square foot?

41. Bought goods as follows: June 1, 1908, on 2 months, \$350; July 15, on 3 months, \$400; Aug. 10, on 4 months, \$450; Sept. 12, on 6 months, \$600. Find the equated time of payment.

42. What rate of discount taken off twice in succession is equivalent to 20 and 25% off?

43. A man owes me \$200 due in 8 months, and \$200 due in 12 months. I accept his note for \$400 due in 10 months. How much do I gain or lose by the transaction, money being worth 8% per annum?

44. What part of the value of a house must be insured at 3% so that in case of loss the owner may receive  $\frac{3}{4}$  of the value, in addition to the premium paid?

45. Find the amount accumulated at the end of 4 years by a person who deposits in a bank at the beginning of each year the sum of \$200, the bank paying 4% interest, compounded half-yearly.

46. What is the cost of boards, at \$1 for 50 sq. ft., to make a closed box 7 ft. 10 in. long, 3 ft. 8 in. wide, 2 ft. 6 in. high (outside dimensions), the boards being 1 inch thick?

47. A farm cost  $3\frac{1}{4}$  times as much as a house; by selling the house at 10% loss and the farm at  $7\frac{1}{7}$ % gain, \$3993.30 is received. Find cost of each.

48. A sum of money at compound interest, added yearly, amounts to \$129600 in 2 years, and to \$178506.25 in 4 years. Find the sum of money and the rate per cent. per annum.

49. A merchant received an invoice of merchandise amounting to \$618 on 4 months' credit, off which he was offered 4% discount for cash. If money is worth 9% per annum, how much cheaper can he get the goods by this offer?

50. A floor 16 ft. by 20 ft. is to be completely covered with carpet 27 inches wide, and having a pattern which matches every 3 feet. If the carpet is to be matched, in which direction must the strips run so as to give the least waste, and how much is the waste?

51. A person bought a quantity of goods for \$224, payable in 2 months, and sold them at once for \$274, payable in 4 months. Find the gain in ready money, if money is worth 6 per cent. per annum.

52. An agent charges the same rate of commission for buying and selling. He sells a consignment for \$8140, deducts \$280 for his commission, and invests the remainder. Find the rate per cent. charged.

53. A person annually increases his capital 20 per cent., less a yearly expenditure of \$500. At the end of four years his capital amounts to \$18052. Find his original capital.

54. What sum should be paid for a \$100 mortgage to run for 5 years at 4 per cent. per annum, in order that the investor may realize 5 per cent. per annum on his outlay?

55. A bankrupt had goods worth \$7950, which, if sold at their full value, would give his creditors  $81\frac{1}{4}$  per cent. of their claims. But  $\frac{3}{5}$  of them were sold at  $17\frac{1}{2}$  per cent. below their value, and the remainder at  $23\frac{3}{4}$  per cent. below their value. How many cents on the dollar did his creditors realize?

56. Find how much a man will plough in 9 hours in a field 40 rods in length, if he walks at the rate of 3 miles an hour, cuts 11 inches wide, and takes 2 minutes turning each time.

57. A person borrows \$500 on April 10th, and on June 22nd pays his debt with \$510.20. At what rate per cent. per annum was he charged interest?

58. A grocer bought a stock of tea, intending to gain 30% on its sale. When he had sold  $\frac{3}{4}$  of it, he was compelled to reduce the price 10 cents a pound, and so gained only  $\frac{8}{9}$  of what he had intended. Find what price he paid for the tea.

59. Bought goods at \$5.70 on 4 months' credit and sold them immediately at \$6.12 on such a term of credit as made my immediate gain  $6\frac{2}{3}\%$ . Reckoning interest at 4% per annum, how long credit did I give?

60. A person makes 7% clear on his money, after paying an income tax of 2 cents on the dollar, by investing in 5% stock. Find the market value of the stock, if a broker charged  $\frac{1}{8}\%$  for transacting the business.

61. A dealer shipped 400 bushels wheat @ \$1.40, 800 bushels @ \$1.62 $\frac{1}{2}$ , and 300 bushels @ \$1.20, to his agent, who sold the first at 20 per cent. gain, the second at 15 per cent. gain, and the third at  $4\frac{1}{8}\%$  per cent. loss; the agent's commission was 3 per cent.; and other charges were \$83.44. Find the dealer's gain per cent.

62. A merchant lost  $6\frac{1}{4}\%$  by marking his goods at a certain per cent. advance on cost, and then giving the same per cent. discount; what was the rate of discount?

63. A rectangular room has its length, breadth and height, as 7, 6, 5, respectively. Its walls were decorated at a cost of 50 cents per square yard, and its ceiling finished at the same rate. The bill for the whole was \$344; what will be the expense of covering the floor with carpet at \$1.40 per square yard?

64. The value of an ounce of the gold of which sovereigns are made is £3 17s. 10 $\frac{1}{2}$ d. What is the weight of 1869 sovereigns?

65. A town borrows \$12000, to be repaid, principal and interest, in 4 equal annual payments. Find the annual payment, money being worth 6% per annum.

66. A grocer wishes to mix 47 pounds of tea worth 40 cents a pound, with a certain quantity worth 35 cents a pound, so that he can sell 16 pounds of the mixture for \$6.07. How many pounds of 35 cent tea must he use?

67. Bought 5 hogsheads of molasses, each containing 84 gallons, at  $12\frac{1}{2}$  cents a gallon, and paid \$25 for freight. Allowing 10% for leakage and waste, 5% of the sales for bad debts, and  $2\frac{1}{2}$ % of the remainder for collecting, for how much a gallon must I sell it to make a gain of 20% on the whole cost?

68. If the cost of an article had been 30% less, the gain per cent. would have been  $2\frac{1}{2}$  times as great. Find the gain per cent.

69. An agent sold a consignment of apples on a commission of 3%. After deducting his commission and reserving a sufficient sum to pay the freight at 20 cents per cwt., he bought flour at \$2.80 per cwt. on a commission of  $2\frac{1}{4}$ %. The total commission was \$63. Find the amount of flour bought.

70. A circular plate of lead 2 inches thick and 8 inches in diameter, is converted without loss into spherical shot of the same density and each .05 inch radius. Find the number of shot.

71. The old par of exchange between the United States and Great Britain was  $\pounds 1 = \$4.44\frac{4}{9}$ ; in 1834 the U.S. Congress reduced the weight of the eagle to 258 grains, and 1837 fixed its fineness at 900 thousandths pure; the mint price of English standard gold (22 carats fine) is  $\pounds 3$  17s.  $10\frac{1}{2}$ d. per ounce. From these facts show the truth of the statement: "By the new par of exchange sterling money is worth  $9\frac{1}{2}$  per cent. more than by the old par."

72. A merchant gives a discount of 10%, but uses a yard measure .72 of an inch too short; what rate of discount would allow him the same amount of gain, if the measure were correct?

73. A sum of money in 2 years, at compound interest, added yearly, amounts to \$1389.15; the present worth of the sum for 1 year is \$1200. Find the rate and the sum.

74. Find how long it will take a train 87 yards long and running at the rate of 40 miles an hour, to pass another one 100 yards long and going in the same direction at the rate of 30 miles an hour.

75. What will be the true interest on \$1000 for 6 months, it being supposed that if this interest is invested for the next six months that the whole interest for the year shall be exactly six per cent?

76. What is the lowest power of  $\cdot 4578$  which is less than  $\cdot 08754$ , the index of the power being a positive integer?

77. Divide 111 into three parts so that the products of each pair may be as 4:5:6.

78. A man has property insured for \$6500, for which he paid \$162.50 premium; he wishes to increase the insurance to \$9000. What extra premium will he be required to pay if the rate for the latter is  $\frac{1}{2}\%$  greater than that for the former?

79. Divide \$2567.50 among A, B, C and D, so that A's share may be to B's as 4 to 5, B's to C's as 6 to 7, and C's to D's as 8 to 9.

80. A merchant discounts with a bank, on Sept. 10th, at the rate of seven per cent. per annum a note for \$950, drawn on that date at three months, bearing interest at eight per cent. per annum. With what amount is he credited?

81. A quantity of sugar, valued at \$42134 Spanish gold, was entered for duty at 30%. In consequence of Spanish gold having been taken at par, whereas it was only worth  $92\frac{1}{2}$  cents on the dollar, a refund of duty was afterwards claimed. Calculate the amount.

82. A rectangular solid is hammered until its length is increased 10 per cent., and its width 15 per cent.; by how much per cent. has its thickness been diminished?

83. If an income tax of 7d. in the pound on all incomes below £100 a year, and of 1s. in the pound on all incomes over £100 a year, realize £18750 on £500000, how much is raised on incomes below £100?

84. A man owes \$3200 due in 7 months; he pays \$1200 in 3 months, \$600 in 5 months, and \$500 in 8 months. How long should the balance run before it becomes due?

85. A grain dealer laid out a certain sum in oats, 25 per cent. more in barley, and in wheat 20 per cent. more than in oats and barley together; he sold the oats at a profit of 8 per cent., the wheat at a profit of 14 per cent., and the barley at a loss of 5 per cent., receiving altogether \$12829.20. Find the amount invested in each kind of grain.

86. At an election the successful candidate received  $\frac{5}{8}$  of the total number of votes cast, and had a majority of 832 over his rival. Of the total number of electors in the constituency  $\frac{3}{18}$  did not vote. How many electors were there in the constituency?

87. Three clocks which tick 70, 80, and 90 times, respectively, a minute, beat coincidently at a certain time; find the least time in which they will again beat together.

88. The power that can raise 33000 lb. through one foot in a minute being taken as the unit, find the measure of the power of an engine that can raise 2000 tons through 50 ft. in 6 hours.

89. A party of men were employed to do a piece of work, which it was necessary to finish in 25 days. When, however, the men had worked 20 days, they had done only  $\frac{5}{7}$  of the work, and 3 more men had then to be employed to complete the work in due time. How many men were employed at first?

90. A merchant buys \$600 worth of sugar, and at first determines to sell it at a profit of 8%, but afterwards decides to add  $\frac{1}{12}$  of its original bulk of sand, and sell it at cost. What does he gain or lose by this method of disposing of it as compared with the former way?

91. The hour and minute hands of a clock are together at 12 o'clock; at what intervals are they together again?

92. There are 50 coins consisting of 25-cent and 20-cent pieces. If each 25-cent piece were a 20-cent piece, and each 20-cent piece a 25-cent piece, the total value of the coins would be \$1 less. How many coins are there of each kind?

93. I want an alloy weighing 1092 ounces, consisting of 13 parts by weight of nickel, 12 parts of lead, and 14 parts of tin. The only nickel I can obtain is contained in an alloy consisting of 14 parts by weight of nickel, 9 parts of lead, and 12 parts of tin. How much of the latter alloy, and of lead and tin must I use?

94. A merchant bought 20 pieces of cloth, each piece containing 25 yards, at \$4.37 $\frac{1}{2}$  per yard, on a credit of 9 months; he immediately sold it at \$4.62 $\frac{1}{2}$ , on a credit of 4 months. What was his actual cash gain, money being worth 6%?

95. If a cask contains 4 parts of wine and 3 parts of water, what fraction of the mixture must be drawn off and water substituted, for the mixture to become 3 parts of wine and 4 parts of water?

96. Three men start together and travel in the same direction. Their rates are 5, 7, and 9 miles per hour respectively. If the course be circular, and 80 rods in circumference, find where and when they will first meet.

97. By what decimal, correct to five places, may any number be multiplied so as to give the same result as dividing that number by 2.302585?

98. Three boats started at the same moment at intervals of 100 yards apart; in 6 minutes the third overtook the second and in two minutes more it overtook the first. How soon will the second overtake the first?

99. A farmer employs a number of men and 8 boys; he pays the boys \$-65 and men \$1-10 each per day. The amount that he paid to all was as much as if each had received \$.92 per day; how many men were employed?

100. The difference between the present worth of a sum of money, due in 2 years, and that of the same sum, due in 4 years, is \$5-10; the difference in the case of 3 and 5 years is \$5-00. Find the rate per cent. per annum.

### EXERCISE V

1. Find the quantities of tea at 25 cents and 37 cents per lb. respectively which must be mixed with 25 lb. at 32 cents per lb. to make up a total of 47 lb., which may be sold at 45 cents per lb., at a gain of 50%.

2. A gallon of water weighs 10 pounds and occupies 277.27 cubic inches; a metre is equal to 39.37 inches; find in grains the weight of a cubic centimetre of water.

3. A silver plate of uniform thickness in the form of a square is worth \$3-20. What is the value of the largest circular disc which can be cut from it?

4. I sent a quantity of goods to a commission merchant with instructions to sell and, after deducting his commission, to invest the proceeds in sugar which is quoted at  $3\frac{3}{8}$  c. per lb. He charges 3% commission for selling and  $2\frac{1}{2}$ % for buying, and his total commissions amount to \$297. How many pounds of sugar did he buy?

5. A man invests \$40000 in a certain stock that sells at a discount of  $20\frac{1}{4}$ % and pays a dividend of  $5\frac{1}{2}$ %; and \$60000 in another stock which sells at a premium of  $19\frac{3}{4}$ % and pays dividends at the rate of  $7\frac{1}{2}$ %. What is his total income if the brokerage in each case be  $\frac{1}{4}$ %?

6. Find the cost in Toronto of a bill of exchange on London for £2769, exchange being quoted at  $8\frac{1}{4}$ .

7. Sound travels at the rate of 1140 ft. per second; how far will a railroad engine running at the rate of 50 miles an hour have gone before the sound of its whistle can be heard 5 miles away?

8. The United States silver dollar weighs  $412\frac{1}{2}$  grains and is  $\frac{9}{10}$  fine; Canadian silver is  $\frac{37}{40}$  fine, and a 5 cent piece weighs 18 grains. What is the ratio of the values of a dollar in U.S. silver, and in Canadian silver?

9. A cubic foot of water weighs 1000 ounces, and the specific gravity of silver is 10.5; find the weight of a solid cone of silver 10 inches in diameter at the base and 12 inches high.

10. A merchant sells two kinds of flour, the superior at \$6 a barrel, the inferior at \$5 a barrel. He sold 150 barrels in all and realized \$810. How many barrels of each kind did he sell?

11. A, B, and C form a trading company for a year, investing respectively \$3000, \$2000, \$1000. A is appointed manager at \$60 a month, and B secretary at \$30 a month, salaries to be reduced in proportion as the capital invested is reduced. At the end of 6 months A withdraws from the business and C is appointed manager, and 2 months later B withdraws and C becomes secretary. How should the year's gain of \$1150 be divided?

12. A sold B a bill of goods, May 7, on 6 month ' credit, amounting to \$2000; July 14, B paid him \$750, and on Sept. 4, \$425. To what additional credit is B entitled?

13. Find the side of a square equal in area to a triangle whose sides are 273, 294, 315.

14. A merchant buys wine at \$1.20 a pint, and dilutes it with water, so that he may make a profit of 20% by selling the mixture at 90 cents a pint. How much water does he add to every gallon of the mixture sold?

15. A can do a certain piece of work in 8 days, working 10 hours a day; B can do the same work in 9 days, working 9 hours a day. They work together and finish the work in 4 days. How many hours a day did they work?

16. The surface of a sphere is 1386 sq. in. and that of a cube is 1536 sq. in. Find which has the greater volume and how much.

17. The Merchants' Bank of New York having declared a dividend of 8%, a stockholder in Buffalo drew on the bank for the sum due him, and sold the draft at a premium of  $1\frac{1}{2}\%$ , thus realizing \$406 from the dividend. How many \$100 shares did he own?

18. A coin consists of 11 parts gold and 1 part alloy, the alloy being worth one-fifteenth of an equal weight of gold. A new coin, of the same weight, is struck, consisting of 9 parts gold and 1 alloy; compare (in the lowest whole numbers) the values of the coins.

19. A commission merchant has goods consigned to him to sell, and, after deducting 2% for both selling and investing the proceeds, he finds that his commission for selling the goods exceeds his commission for buying by \$12. Find the value of the goods remitted to him.

20. The compound interest on a certain sum for the second year is \$49.92, and for the third year \$51.9168. Find the sum and the rate per cent., compounded yearly.

21. A mortgage for \$1000, bearing interest at the rate of 8 per cent. per annum, payable half-yearly, has two years to run. Find its present value if money is worth six per cent. per annum.

22. The area of a rhombus is 990 sq. yd., and the length of one of its diagonals is 55 yd. Find the other diagonal.

23. A man bought a house for \$2375, and rented it for \$18 per month. He paid \$47 a year for taxes and repairs, and insured the house to cover its cost at  $\frac{4}{5}\%$  annually. Find his net annual income, and the rate of interest to which it is equivalent on the money invested.

24. If 1700 metres are equal to 1 mile, and if a cubic foot of water weighs 1000 ounces avoirdupois, and a cubic metre of water 1000 kilogrammes, find the ratio of a kilogramme to a pound troy.

25. If the area of a postage stamp ( $\frac{3}{4}$  in. by  $\frac{7}{8}$  in.) be taken as the unit, find the measure of the area of an envelope  $3\frac{1}{8}$  in. by  $5\frac{1}{2}$  in.

26. A and B can do a piece of work in 4 days; B and C in 6 days; C alone in 10 days. How long would it take A and C together to do it?

27. The par of exchange between Paris and London is 25·2215 francs for £1, and between St. Petersburg and London 38·177 pence for 1 rouble. Find the par of exchange between Paris and St. Petersburg.

28. A note for \$182·50 was discounted at a bank on March 24th at the rate of 7%; it fell due on July 2nd following, and was redeemed by the maker. What rate of interest did the bank make on its investment?

29. A conical tent whose slant height is 12 ft. requires 132 sq. ft. of canvas to make it; how much ground does the floor of the tent cover?

30. A cask contains a mixture of wine and water, in the ratio of 5 to 1. How much of the mixture must be drawn off and water substituted for it, so that the mixture in the cask may contain wine and water in the ratio of 3 to 2?

31. A merchant bought 180 pounds of tea, and 560 pounds of coffee, the coffee costing  $\frac{8}{15}$  as much per lb. as the tea. He sold the coffee at an advance of 50% and the tea at a loss of 25%, gaining on the whole \$62·60. What did the tea cost per pound?

32. A commission merchant sells goods for an employer, and, after deducting his commissions, invests the net amount in other goods. His total commission is \$100, receiving \$2 more for selling than for buying. If his rate for selling is the same as the rate for buying, what is that rate?

33. I bought goods for \$30.40 on 4 months' credit, and sold them immediately for \$37.20, with such allowance of credit as made my immediate gain 20%. How long credit did I give, money being worth 4% per annum?

34. Reckoning a pint to be 30 cub. in.; if 462 gal. are taken out of a cylindrical cistern 7 ft. in diameter, how many inches will the surface of the water be lowered? ( $\pi=3\frac{1}{7}$ .)

35. A farmer has sheep worth \$4, \$5, \$6, \$8 per head. What number may he sell of each to realize an average price of \$5.50 per head?

36. A merchant in London, England, consigns to his agent in Montreal 5000 yards of cloth invoiced at 6 shillings a yard, and instructs him to sell the cloth and invest the proceeds in apples, after deducting the following charges:—Commissions 5% each, freight \$325 and an ad valorem duty of 25%. The cloth is sold at \$2 a yard and apples are bought at \$2 a barrel. Find the merchant's gain if the apples net him in London 10 shillings a barrel, sterling exchange at par.

37. A man invested a certain sum in 3 per cent. stock at 75 and another sum greater by \$3000 in 5 per cent. stock at 120. If the income from the latter exceeds that from the former by \$134, find the sums invested.

38. If an ordinary brick measures  $8'' \times 4'' \times 2''$ , and 81000 such bricks are piled in the form of a rectangular solid whose length, width and height are proportional to 4, 3 and 2, find the dimensions of the pile.

39. How much money deposited in a bank will amount to \$1500 in 1 year, the bank paying 3% per annum, compounded quarterly?

40. There are two rectangles, one 189 yd. by 45 yd., the other 244 yd. by 36 yd. Show that there is only one square whose area is less than that of the one rectangle and greater than that of the other, and whose side is an integral number of yards in length; and find the length of its side.

41. A man rows 3 miles down a stream in 40 minutes; without the aid of the stream it would take him an hour. How long would it take him to return the 3 miles against the stream?

42. Find a merchant's gain, through dishonesty, if he sell goods, which cost him \$84, by a false pound weight,  $\frac{1}{4}$  ounce too light.

43. A train starts from A at 6 o'clock a.m. and reaches B at 2 o'clock p.m.; another train starts from B at 11 a.m. and reaches A at 5 p.m. At what time of the day do they meet?

44. Calculate to within  $\frac{1}{1000}$  of one per cent. the rate per cent. that paid quarterly is equivalent to 8 per cent. paid annually.

45. A stock company with a capital of \$100000, 10% of which is paid up, can declare a dividend of 12% on the paid-up capital. They make a further call of 4%, which is devoted to increasing their office accommodation; what rate of dividend can they now declare?

46. I borrowed \$600 on June 30th, 1904, agreeing to pay  $7\frac{1}{2}\%$  interest on the principal, reserving the right to pay off any part of the principal when the interest is paid up to date. On Sept. 11th, 1904, I paid \$200; on June 30th, 1905, I paid \$150; what payment made on Jan. 31st, 1907, cancelled the debt?

47. A closed rectangular vessel, formed of metal 1 inch thick, whose external dimensions are 12 inches, 10 inches, and 8 inches, weighs 89 pounds. What would be the weight of a solid mass of metal of the same dimensions?

48. By selling 40 yd. of cotton for a certain sum of money a merchant gains 25%; how many yards will he give for the same sum if he decides to advance his profit to 50%?

49. When it is 7.15 p.m. at Greenwich, what o'clock is it at Madras, the longitude of which is  $80^\circ$  east?

50. If a train 88 yards long overtake a person, walking at the rate of 4 miles per hour along the railway, and pass him in 8 seconds, what is the rate of the train in miles per hour?

51. A merchant is paid \$8.64 for a coat; the manufacturer, the wholesaler and the retailer have made the same rate of profit; the cost to the manufacturer was \$5; what was the rate of profit?

52. What yearly rate of interest is equivalent to 4% per annum compounded quarterly?

53. An agent received a consignment of lumber which he sold, charging a commission of 3 per cent. With the net proceeds after deducting a commission of 2 per cent. for purchasing, he bought 9700 bushels of wheat at 80 cents a bushel. Find the value of the lumber sold.

54. A Toronto merchant owes 18000 francs in Paris. He buys a draft on London when sterling exchange is at 8, and when exchange between London and Paris is 25·20 francs per £. What does he pay for the draft?

55. The cost of inclosing a circular field of 10 acres by a fence is \$30. Find the cost of inclosing a circular field of  $22\frac{1}{2}$  acres by a similar fence.

56. A person borrows \$2560, and at the end of each year pays \$650 to reduce the principal and to pay interest at the rate of 5% on the sum which has been standing against him through the year. How much will remain of the debt at the end of three years?

57. A person bought 30 pounds of sugar, of two different kinds, for \$2·28; the better at 10 cents a pound, and the inferior at 7 cents a pound. How many pounds of each did he buy?

58. A note for \$960·50, drawn May 23 at 60 days, is discounted June 6 at 8 per cent. Find the proceeds of the note and the rate of interest received by the bank on the money advanced.

59. Five long-distance riders go round a circular track 8, 9, 10, 11 and 12 times respectively in one hour. If all start together when will all be together again? What is the least number of minutes start which the fastest man must give each of the others that all may finish together in a race of 20 times round the track?

60. Two pipes can fill a cistern in 30 and 35 minutes, respectively. When the cistern is empty both pipes are opened together, but at the end of 12 minutes the second is turned off. How long does it take to fill the cistern?

61. I owe a man \$850, and give him my note at 90 days. What must be the face of the note to pay the exact amount, if discounted at 6% per annum?

62. What will it cost a Toronto merchant for a draft on London for £370 17s. 6d., when sterling exchange is quoted at  $8\frac{1}{4}$ ?

63. An agent, A, insures a cargo for \$80000 at  $\frac{1}{4}\%$ . B takes half A's risk at  $\frac{3}{8}\%$  and C half of the remainder at  $\frac{5}{8}\%$ , while D takes  $\frac{1}{2}$  B's risk at  $\frac{1}{2}\%$ . In case the ship is safe find the profit or loss of each agent?

64. A sold 2 city lots, which cost the same price, to B at a loss of 15%; B sold them to C, gaining 20% on one, and losing 25% on the other. What did either lot cost A, if B received \$153 more for one than for the other?

65. If the side of a square lot containing one-tenth of an acre is expressed by the number 11, find the unit of linear measurement.

66. A merchant makes a purchase of cloth, marks it at an advance of 20 per cent. on cost, and, after selling one-half of it, finds that one-third of the remainder is so damaged as to sell for only one-half of its cost. What advance must be made on the marked price of what now remains so that on the whole there may be a gain of 15 per cent?

67. From the fact that 4% standard stocks are selling at 93, calculate the sum that should be paid at the present time to discharge a debt of \$4335 due ten months hence.

68. There are two places 126 miles apart, from which two persons start at the same time, with the design of meeting; one travels at the rate of 3 miles in 2 hours, and the other at the rate of 5 miles in 4 hours. When will they meet?

69. Find (a) the total surface, (b) the volume, of a block of wood 18 inches square and 3 inches thick, with a circular hole of 14 inches diameter through its centre.

70. A merchant in London (Eng.) owed another in St. Petersburg 9842 roubles, which he remitted through Paris, when the exchange was 25.35 francs for £1, and between Paris and St. Petersburg 3.39 francs for 1 rouble. Shortly after, the exchange between London and Paris was 25.625 francs for £1, and between Paris and St. Petersburg 3.37 francs for 1 rouble. How much would he have gained by the delay?

71. A note for \$1000 due 73 days hence is discounted at a bank at 8% per annum. If the banker has to pay  $3\frac{1}{2}\%$  interest on any money he uses, find his net profit on this transaction.

72. A commission merchant sold 2 lots for \$800, on one of them he charged 2% commission for selling, and on the other 4%; his total commission being \$26, find the selling price of each of the lots.

73. \$15000 is deposited in a Savings Bank and allowed to accumulate interest at the rate of 4% per annum for 3 years. What annual income would a person derive thereafter from the interest (at 4% per annum) on the sum thus accumulated?

74. What length of wire, 0.1 inch in diameter, can be made from a cubic foot of metal?

75. A mixture of black and green tea, weighing 14 pounds, cost \$5.12 $\frac{1}{2}$ . If the proportions are interchanged the mixture would be worth \$5.02 $\frac{1}{2}$ . The black tea is worth 37 $\frac{1}{2}$  cents a pound. Find the price per pound of the green tea.

76. A merchant imports goods, which cost him in England £720. He pays an ad valorem duty of 10%, and sells them through an agent for \$4200. If the agent charges 5% for selling them, find his gain (exchange being at par).

77. A sum of money together with the interest which accumulates during the time will pay the wages of 17 men for 4 days, or 12 men for 6 days. How many men in either case does the interest pay?

78. A square field, containing 16 ac. 401 sq. yd., has a walk around it outside 12 ft. in width. Find the area of the walk in yards.

79. In running a 10-mile race, on a quarter mile track, A overlaps B for the first time at the end of the 34th round. By what distance will A win at the same rate of running?

80. What rate of trade discount deducted 4 times in succession is equivalent to  $59\frac{1}{5}\%$  off?

81. A square inclosure has a side 40 feet in length. In front of it, and at a distance of 40 feet from each of the two nearest corners, a cow is tethered. If the tether rope is 100 feet in length, and the cow is unable to enter the inclosure, find the area of the ground from which the cow will be able to procure the grass.

82. The cost of fencing a circular plot of ground at \$1.25 a yd was \$715.00. Find the length of a straight path running from side to side through the centre

83. A trader bought merchandise as follows: July 3rd, \$35.26; July 4th, \$48.65, on 30 days, August 17th, \$6.48, September 12th, \$50. What is due on account on October 12th (interest at 9%)?

84. A merchant buys a quantity of coal oil for \$1500; by the use of a false measure, and by marking his goods at a profit of 25%, he gains \$750. Find the size of his gallon measure.

85. Compare the areas of the two segments of a circle made by a chord equal to the radius ( $\pi=3.1416$ ).

86. What per cent. advance on cost must a merchant mark his goods, so that, after allowing 5% of the sales for bad debts, an average credit of 6 months, and 7% of the prime cost of the goods for expenses, he may make a clear gain of  $12\frac{1}{2}\%$  on the cost, money being worth 6%?

87. A person shooting at a target, at a distance of 545 yards, hears the bullet strike the target 4 seconds after he fired. A spectator, equally distant from the target and the shooting point, hears the shot strike  $2\frac{1}{2}$  seconds after he heard the report. Find the velocity of sound.

88. A military tent is 9 feet high; its shape is that of a cone standing on a cylinder whose diameter is 12 feet and height 3 feet. Find the cost of the canvas, 27 inches wide, at 15c. a yard, allowing 3 yards for seams and waste.

89. A merchant in Montreal shipped to London goods on which he paid \$262.75 for freight and storage. His agent charged him a commission of 5 per cent. for selling, and the goods realized £750 in London. The merchant's net profit was  $33\frac{1}{3}$  per cent. Find what the goods originally cost him, the rate of exchange between London and Montreal being £1 for \$4.86.

90. The longitude of Tokio is  $140^\circ$  east, that of New York  $74^\circ$  west. What is the time at Tokio, when it is 5.10 p.m. on October 7th in New York?

91. The base of a prism of height 125 inches is a parallelogram with a diagonal 104 inches and two sides 45 inches and 85 inches. Find the volume.

92. If 5 acres of grass, together with what grows during the time of grazing, keep 20 oxen 10 weeks, and 8 acres keep 29 oxen 16 weeks, how many weeks will 15 acres keep 70 oxen?

93. The difference between the compound interest on a certain sum, for the first and second years, is \$2.40; the difference for the second and third years is \$2.496. Find the sum and the rate per cent., compounded yearly.

94. A rectangular field, whose width is  $\frac{3}{4}$  of its length, contains 15 acres 123 sq. rods. In going from one corner to the opposite how much shorter is it to take the diagonal than to go around the two sides?

95. An article which cost \$96 was sold for \$90; it had been marked at a certain advance on cost, but the same rate of discount was given, what was the marked price?

96. What is the present value of a mortgage of \$5000, bearing interest at 6%, payable yearly, having 5 years and 6 months to run, the next payment of interest being due in 6 months, money being worth 5% per annum, payable yearly?

97. A dealer purchased on 6 months' credit goods to the amount of \$520.20; after keeping them 3 months he sold them on credit for \$575.96, and allowing money to be worth 8%, he found that he had made 10 per cent. on the transaction. On what term of credit did he sell the goods?

98. Water is flowing at the rate of 10 miles per hour through a pipe 14 in. in diameter, into a rectangular reservoir 187 yd. by 96 yd. In what time will the surface be raised 1 inch?

99. A merchant bought 777 yards of cloth at \$1.50 per yard. In selling the first half he uses a 35 inch yard measure, and in addition had the goods marked at an advance of 25% on the cost. At what advance per cent. on the cost should he mark the remainder so as to gain only 25% on the whole outlay, if he use a 37 inch yard measure in selling it?

100. A merchant in Vancouver owes \$5000 in Halifax. Which of the following ways of cancelling the debt is the more advantageous, and by how much: (a) to send to Halifax a draft on Halifax purchased at  $\frac{1}{2}\%$  premium, or (b) to send to Halifax a draft on Vancouver which sells in Halifax at  $\frac{1}{2}\%$  discount?

**INTEREST TABLES**

**TABLES OF WEIGHTS AND MEASURES**

**THE METRIC SYSTEM OF MEASUREMENT**

**DEFINITIONS OF ARITHMETICAL TERMS**

**AMOUNT of \$1 at interest, compounded yearly,  
for a given number of years**

Yrs.	1%	1½%	2%	2½%	3%	3½%	Yrs.
1	1.01000	1.01500	1.02000	1.02500	1.03000	1.03500	1
2	1.02010	1.03023	1.04040	1.05063	1.06090	1.07123	2
3	1.03030	1.04568	1.06121	1.07689	1.09273	1.10872	3
4	1.04060	1.06136	1.08243	1.10381	1.12551	1.14752	4
5	1.05101	1.07728	1.10408	1.13141	1.15927	1.18769	5
6	1.06152	1.09344	1.12616	1.15969	1.19405	1.22926	6
7	1.07214	1.10985	1.14869	1.18869	1.22987	1.27228	7
8	1.08286	1.12649	1.17166	1.21840	1.26677	1.31681	8
9	1.09369	1.14339	1.19509	1.24886	1.30477	1.36290	9
10	1.10462	1.16054	1.21899	1.28008	1.34392	1.41060	10
11	1.11567	1.17795	1.24337	1.31209	1.38423	1.45997	11
12	1.12683	1.19562	1.26824	1.34489	1.42576	1.51107	12
13	1.13809	1.21355	1.29361	1.37851	1.46853	1.56396	13
14	1.14947	1.23176	1.31948	1.41297	1.51259	1.61869	14
15	1.16097	1.25023	1.34587	1.44830	1.55797	1.67535	15
16	1.17258	1.26899	1.37279	1.48451	1.60471	1.73399	16
17	1.18430	1.28802	1.40024	1.52162	1.65285	1.79468	17
18	1.19615	1.30734	1.42825	1.55966	1.70243	1.85749	18
19	1.20811	1.32695	1.45681	1.59865	1.75351	1.92250	19
20	1.22019	1.34686	1.48595	1.63862	1.80611	1.98979	20
21	1.23239	1.36706	1.51567	1.67958	1.86029	2.05943	21
22	1.24472	1.38756	1.54598	1.72157	1.91610	2.13151	22
23	1.25716	1.40838	1.57690	1.76461	1.97359	2.20611	23
24	1.26973	1.42950	1.60844	1.80873	2.03279	2.28333	24
25	1.28243	1.45095	1.64060	1.85394	2.09378	2.36324	25
26	1.29526	1.47271	1.67342	1.90029	2.15659	2.44596	26
27	1.30821	1.49480	1.70689	1.94780	2.22129	2.53157	27
28	1.32129	1.51722	1.74102	1.99650	2.28793	2.62017	28
29	1.33450	1.53998	1.77584	2.04641	2.35657	2.71188	29
30	1.34785	1.56308	1.81136	2.09757	2.42726	2.80679	30
31	1.36133	1.58653	1.84759	2.15001	2.50008	2.90503	31
32	1.37494	1.61032	1.88454	2.20376	2.57508	3.00671	32
33	1.38869	1.63448	1.92223	2.25885	2.65234	3.11194	33
34	1.40258	1.65900	1.96068	2.31532	2.73191	3.22086	34
35	1.41660	1.68388	1.99988	2.37320	2.81386	3.33359	35
36	1.43077	1.70914	2.03988	2.43253	2.89827	3.45026	36
37	1.44508	1.73478	2.08068	2.49334	2.98522	3.57102	37
38	1.45953	1.76080	2.12229	2.55568	3.07478	3.69601	38
39	1.47412	1.78721	2.16474	2.61957	3.16702	3.82537	39
40	1.48886	1.81402	2.20803	2.68506	3.26203	3.95925	40

## AMOUNT of \$1—Continued

Yrs.	4%	4½%	5%	6%	7%	8%	Yrs.
1	1.04000	1.04500	1.05000	1.06000	1.07000	1.08000	1
2	1.08160	1.09203	1.10250	1.12360	1.14490	1.16640	2
3	1.12486	1.14117	1.15763	1.19102	1.22504	1.25971	3
4	1.16986	1.19252	1.21551	1.26248	1.31080	1.36049	4
5	1.21665	1.24618	1.27628	1.33823	1.40255	1.46933	5
6	1.26532	1.30226	1.34010	1.41852	1.50073	1.58687	6
7	1.31593	1.36086	1.40710	1.50363	1.60578	1.71382	7
8	1.36857	1.42210	1.47746	1.59385	1.71819	1.85093	8
9	1.42331	1.48610	1.55133	1.68948	1.83846	1.99900	9
10	1.48024	1.55297	1.62889	1.79085	1.96715	2.15893	10
11	1.53945	1.62285	1.71034	1.89830	2.10485	2.33164	11
12	1.60103	1.69588	1.79586	2.01220	2.25219	2.51817	12
13	1.66507	1.77220	1.88565	2.13293	2.40985	2.71962	13
14	1.73168	1.85194	1.97993	2.26090	2.57853	2.93719	14
15	1.80094	1.93528	2.07893	2.39656	2.75903	3.17217	15
16	1.87298	2.02237	2.18287	2.54035	2.95216	3.42594	16
17	1.94790	2.11338	2.29202	2.69277	3.15882	3.70002	17
18	2.02582	2.20848	2.40662	2.85434	3.37993	3.99602	18
19	2.10685	2.30786	2.52695	3.02560	3.61653	4.31570	19
20	2.19112	2.41171	2.65330	3.20714	3.86968	4.66096	20
21	2.27877	2.52024	2.78596	3.39956	4.14056	5.03383	21
22	2.36992	2.63365	2.92526	3.60354	4.43040	5.43654	22
23	2.46472	2.75217	3.07152	3.81975	4.74053	5.87146	23
24	2.56330	2.87601	3.22510	4.04893	5.07237	6.34118	24
25	2.66584	3.00543	3.38635	4.29187	5.42743	6.84848	25
26	2.77247	3.14068	3.55567	4.54938	5.80735	7.39635	26
27	2.88337	3.28201	3.73346	4.82235	6.21387	7.98806	27
28	2.99870	3.42970	3.92013	5.11169	6.64884	8.62711	28
29	3.11865	3.58404	4.11614	5.41839	7.11426	9.31727	29
30	3.24340	3.74532	4.32194	5.74349	7.61226	10.06266	30
31	3.37313	3.91386	4.53804	6.08810	8.14511	10.86767	31
32	3.50806	4.08998	4.76494	6.45339	8.71527	11.73708	32
33	3.64838	4.27403	5.00319	6.84059	9.32534	12.67605	33
34	3.79432	4.46636	5.25335	7.25103	9.97811	13.69013	34
35	3.94608	4.66734	5.51601	7.68609	10.67658	14.78534	35
36	4.10393	4.87737	5.79181	8.14725	11.42394	15.96817	36
37	4.26808	5.09686	6.08140	8.63609	12.22362	17.24562	37
38	4.43881	5.32621	6.38547	9.15425	13.07927	18.62527	38
39	4.61636	5.56589	6.70475	9.70351	13.99482	20.11529	39
40	4.80102	5.81636	7.03998	10.28572	14.97446	21.72452	40

**PRESENT VALUE of \$1, due at the end of a given  
number of years, interest compounded yearly**

Yrs.	1%	1½%	2%	2½%	3%	3½%	Yrs.
1	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	1
2	0.98030	0.97066	0.96117	0.95181	0.94260	0.93351	2
3	0.97059	0.95632	0.94232	0.92860	0.91514	0.90194	3
4	0.96098	0.94218	0.92385	0.90595	0.88849	0.87144	4
5	0.95147	0.92826	0.90573	0.88385	0.86261	0.84197	5
6	0.94204	0.91454	0.88797	0.86230	0.83748	0.81350	6
7	0.93272	0.90103	0.87056	0.84127	0.81309	0.78599	7
8	0.92348	0.88771	0.85349	0.82075	0.78941	0.75941	8
9	0.91434	0.87459	0.83675	0.80073	0.76642	0.73373	9
10	0.90529	0.86167	0.82035	0.78120	0.74409	0.70892	10
11	0.89632	0.84893	0.80426	0.76214	0.72242	0.68495	11
12	0.88745	0.83639	0.78849	0.74356	0.70138	0.66178	12
13	0.87866	0.82403	0.77303	0.72542	0.68095	0.63940	13
14	0.86996	0.81185	0.75788	0.70773	0.66112	0.61778	14
15	0.86135	0.79985	0.74302	0.69047	0.64186	0.59689	15
16	0.85282	0.78803	0.72845	0.67363	0.62317	0.57671	16
17	0.84438	0.77639	0.71416	0.65720	0.60502	0.55720	17
18	0.83602	0.76491	0.70016	0.64117	0.58739	0.53836	18
19	0.82774	0.75361	0.68643	0.62553	0.57029	0.52016	19
20	0.81954	0.74247	0.67297	0.61027	0.55368	0.50257	20
21	0.81143	0.73150	0.65978	0.59539	0.53755	0.48557	21
22	0.80340	0.72069	0.64684	0.58087	0.52189	0.46915	22
23	0.79544	0.71004	0.63416	0.56670	0.50669	0.45329	23
24	0.78757	0.69954	0.62172	0.55288	0.49193	0.43796	24
25	0.77977	0.68921	0.60953	0.53939	0.47761	0.42315	25
26	0.77205	0.67902	0.59758	0.52624	0.46369	0.40884	26
27	0.76440	0.66899	0.58586	0.51340	0.45019	0.39501	27
28	0.75684	0.65910	0.57438	0.50088	0.43708	0.38165	28
29	0.74934	0.64936	0.56311	0.48866	0.42435	0.36875	29
30	0.74192	0.63976	0.55207	0.47674	0.41199	0.35628	30
31	0.73458	0.63031	0.54125	0.46512	0.39999	0.34423	31
32	0.72730	0.62099	0.53063	0.45377	0.38834	0.33259	32
33	0.72010	0.61182	0.52023	0.44270	0.37703	0.32134	33
34	0.71297	0.60277	0.51003	0.43191	0.36605	0.31048	34
35	0.70591	0.59387	0.50003	0.42137	0.35538	0.29998	35
36	0.69893	0.58509	0.49022	0.41109	0.34503	0.28983	36
37	0.69201	0.57644	0.48061	0.40107	0.33498	0.28003	37
38	0.68515	0.56792	0.47119	0.39129	0.32523	0.27057	38
39	0.67837	0.55953	0.46195	0.38174	0.31575	0.26141	39
40	0.67165	0.55126	0.45289	0.37243	0.30656	0.25257	40

## PRESENT VALUE of \$1—Continued

Yrs.	4%	4½%	5%	6%	7%	8%	Yrs.
1	0.96154	0.95694	0.95238	0.94340	0.93458	0.92593	1
2	0.92456	0.91573	0.90703	0.89000	0.87344	0.85734	2
3	0.88900	0.87630	0.86384	0.83962	0.81630	0.79383	3
4	0.85480	0.83856	0.82270	0.79209	0.76290	0.73503	4
5	0.82193	0.80245	0.78353	0.74726	0.71299	0.68058	5
6	0.79031	0.76790	0.74622	0.70496	0.66634	0.63017	6
7	0.75992	0.73483	0.71068	0.66506	0.62275	0.58349	7
8	0.73069	0.70319	0.67684	0.62741	0.58201	0.54027	8
9	0.70259	0.67290	0.64461	0.59190	0.54393	0.50025	9
10	0.67556	0.64393	0.61391	0.55840	0.50835	0.46319	10
11	0.64958	0.61620	0.58468	0.52679	0.47509	0.42888	11
12	0.62460	0.58966	0.55684	0.49697	0.44401	0.39711	12
13	0.60057	0.56427	0.53032	0.46884	0.41496	0.36770	13
14	0.57748	0.53997	0.50507	0.44230	0.38782	0.34046	14
15	0.55526	0.51672	0.48102	0.41727	0.36245	0.31524	15
16	0.53391	0.49447	0.45811	0.39365	0.33874	0.29189	16
17	0.51337	0.47318	0.43630	0.37136	0.31657	0.27027	17
18	0.49363	0.45280	0.41552	0.35034	0.29586	0.25025	18
19	0.47464	0.43330	0.39573	0.33051	0.27651	0.23171	19
20	0.45639	0.41464	0.37689	0.31181	0.25842	0.21455	20
21	0.43883	0.39679	0.35894	0.29416	0.24151	0.19866	21
22	0.42196	0.37970	0.34185	0.27751	0.22571	0.18394	22
23	0.40573	0.36335	0.32557	0.26180	0.21095	0.17032	23
24	0.39012	0.34770	0.31007	0.24698	0.19715	0.15770	24
25	0.37512	0.33273	0.29530	0.23300	0.18425	0.14602	25
26	0.36069	0.31840	0.28124	0.21981	0.17220	0.13520	26
27	0.34682	0.30469	0.26785	0.20737	0.16093	0.12519	27
28	0.33348	0.29157	0.25509	0.19563	0.15040	0.11591	28
29	0.32065	0.27902	0.24295	0.18456	0.14056	0.10733	29
30	0.30832	0.26700	0.23138	0.17411	0.13137	0.09938	30
31	0.29646	0.25550	0.22036	0.16426	0.12277	0.09202	31
32	0.28506	0.24450	0.20987	0.15496	0.11474	0.08520	32
33	0.27409	0.23397	0.19987	0.14619	0.10724	0.07889	33
34	0.26355	0.22390	0.19036	0.13791	0.10022	0.07305	34
35	0.25341	0.21425	0.18129	0.13011	0.09366	0.06763	35
36	0.24367	0.20503	0.17266	0.12274	0.08754	0.06263	36
37	0.23430	0.19620	0.16444	0.11579	0.08181	0.05799	37
38	0.22529	0.18775	0.15661	0.10924	0.07646	0.05369	38
39	0.21662	0.17967	0.14915	0.10306	0.07146	0.04971	39
40	0.20829	0.17193	0.14205	0.09722	0.06678	0.04603	40

## TABLES OF WEIGHTS AND MEASURES

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### LENGTH

The standard unit for the measurement of length is the YARD.

1 yard = 3 feet = 36 inches.

$5\frac{1}{2}$  yards = 1 rod (pole or perch).

1760 yards = 1 mile = 320 rods.

22 yards = 1 chain = 100 links.

220 yards = 1 furlong = 40 rods.

2 yards = 1 fathom.

Also, 4 inches = 1 hand, used in measuring the height of horses.

### AREA

A square foot = 144 square inches.

A square yard = 9 square feet.

A square rod =  $30\frac{1}{4}$  square yards.

An acre = 10 square chains.

= 160 square rods.

= 4840 square yards.

A square mile = 640 acres.

Also, 100 square feet = 1 square, used in measuring roofing, flooring, etc.

### VOLUME

A cubic yard = 27 cubic feet.

A cubic foot = 1728 cubic inches.

A cord (of firewood, etc.) = 128 cubic feet.

## CAPACITY

The standard unit for the measure of capacity is the GALLON, which is the space occupied by ten pounds of distilled water at a temperature of 62° F.

1 gallon = 4 quarts = 8 pints = 32 gills.

2 gallons = 1 peck.

8 gallons = 1 bushel.

31½ gallons = 1 barrel.

63 gallons = 1 hogshead.

Also, 8 bushels = 1 quarter, used in reporting the British grain market.

For Apothecaries' Fluid Measure the pint is sub-divided as follows:

1 pint = 20 fluid ounces.

1 fluid ounce = 8 fluid drachms.

1 fluid drachm = 60 minims.

## WEIGHT

The standard unit for the measurement of weight is the POUND AVOIRDUPOIS.

1 pound = 16 ounces = 7000 grains.

100 pounds = 1 cental.

2000 pounds = 1 ton.

196 pounds = 1 barrel of flour.

200 pounds = 1 barrel of beef or of pork.

280 pounds = 1 barrel of salt.

An ounce Troy = 480 grains.

A pound Troy = 12 Troy ounces.

= 5760 grains.

In contracts for the sale and delivery of any of the undermentioned articles, the bushel shall be determined by weighing, unless a bushel by measure is specially agreed upon, the weight equivalent to a bushel being as follows:

14 lb. of Blue Grass Seed.	56 lb. of Indian Corn, Rye,
34 " Oats.	or Flax Seed.
36 " Malt.	60 " Beans, Beets, Car-
40 " Castor Beans.	rots, Parsnips,
44 " Hemp Seed.	Peas, Potatoes,
48 " Barley, Buck-	Clover Seed, Tur-
wheat or Timo-	nips or Wheat.
thy Seed.	70 " Bituminous Coal.
50 " Onions.	80 " Lime.

## VALUE

## 1. Canadian and United States money.

The standard unit of value is the DOLLAR, which is the value of about  $23\frac{1}{3}$  grains of pure gold.

$$1 \text{ dollar} = 100 \text{ cents} = 1000 \text{ mills.}$$

In the United States there is the eagle (=10 dollars), and also the dime (=10 cents); but these denominations are not regarded in business operations, so that the table of U.S. money is practically the same as our own.

## 2. British money.

$$4 \text{ farthings} = 1 \text{ penny.}$$

$$12 \text{ pence} = 1 \text{ shilling.}$$

$$20 \text{ shillings} = 1 \text{ pound (the unit).}$$

$$21 \text{ shillings} = 1 \text{ guinea.}$$

The units in British and Canadian money are connected as follows:

$$\$73 = \pounds 15, \text{ or, } \$4.86\frac{2}{3} = \pounds 1.$$

## TIME

The unit for the measurement of time is the MEAN SOLAR DAY.

$$1 \text{ day} = 24 \text{ hours.}$$

$$1 \text{ hour} = 60 \text{ minutes.}$$

$$1 \text{ min.} = 60 \text{ seconds.}$$

$$7 \text{ days} = 1 \text{ week.}$$

$$365 \text{ days} = 1 \text{ common year.}$$

$$366 \text{ days} = 1 \text{ leap year.}$$

The calendar year is divided into 12 months of different lengths, as follows:

January,	31 days.	July,	31 days.
February,	28 "	August,	31 "
March,	31 "	September,	30 "
April,	30 "	October,	31 "
May,	31 "	November,	30 "
June,	30 "	December,	31 "

In leap year February has 29 days, making 366 days in the whole year.

To ascertain which is leap year, divide the number denoting the year by 4; if there is no remainder the number denotes leap year, while remainders 1, 2, 3, indicate respectively the 1st, 2nd and 3rd year after leap year. Thus, 1891 was the 3rd year after leap year, and 1912 and 1916 will be leap years.

If the number denoting the year ends in two ciphers it must be divisible by 400 in order to indicate leap year; thus, the years 2000 and 2400 will be leap years, but the year 1900 was not a leap year.

### ANGLES

The circumference of a circle is divided into 360 equal parts, and each part subtends at the centre an angle called a degree, each degree is divided into 60 equal parts called minutes, and each minute into 60 seconds.

1 degree = 60 minutes = 3600 seconds.  
90 degrees = 1 quadrant or right angle.

### MISCELLANEOUS

12 articles = 1 dozen.  
12 dozen = 1 gross.  
24 sheets of paper = 1 quire.  
20 quires = 1 ream.

## METRIC SYSTEM OF MEASUREMENT

### LENGTH

The unit of length is the METRE (= 39.37 inches nearly). To express fractions and multiples of the unit the following significant prefixes are used:

micro denoting	....	$\frac{1}{1000000}$	....	abbreviated	$\mu$ .
milli	"	....	$\frac{1}{1000}$	....	" m.
centi	"	....	$\frac{1}{100}$	....	" c.
deci	"	....	$\frac{1}{10}$	....	" d.
deka	"	....	10	....	" D.
hecto	"	....	100	....	" H.
kilo	"	....	1000	....	" K.
myria	"	....	10000	....	" M.
mega	"	....	1000000	....	" Mg.

1 metre =	1000000	micrometres, or microns,	$\mu$ m.
1 metre =	1000	millimetres,	mm.
1 metre =	100	centimetres,	cm.
1 metre =	10	decimetres,	dm.
10 metres =	1	dekametre,	Dm.
100 metres =	1	hectometre,	Hm.
1000 metres =	1	kilometre,	Km.
10000 metres =	1	myriametre,	Mm.
1000000 metres =	1	megametre,	Mgm.

### AREA

The unit of area is the ARE—equivalent to one square dekametre.

1 are =	1000	milliares,	ma.
1 are =	100	centiares,	ca.
1 are =	10	deciars,	da.
10 ares =	1	dekare,	Da.
100 ares =	1	hectare,	Ha.

### VOLUME

One unit of volume is the STERE—equivalent to one cubic metre; it is used in measuring wood, excavations, etc.

1 stere	= 100 centisteres,	cs.
1 stere	= 10 decisteres,	ds.
10 steres	= 1 dekastere,	Ds.
100 steres	= 1 hectostere,	Hs.
1000 steres	= 1 kilostere,	Ks.

Another unit of volume is the LITRE—equivalent to one cubic decimetre; it is used for fluid measure, or measure of capacity.

1 litre	= 100 centilitres,	cl.
1 litre	= 10 decilitres,	dl.
10 litres	= 1 dekalitre,	Dl.
100 litres	= 1 hectolitre,	Hl.
1000 litres	= 1 kilolitre,	Kl.
	= 1 stere.	

### WEIGHT

The unit of weight (or mass) is the GRAMME—equivalent to the weight of one cubic centimetre of distilled water at its temperature of maximum density.

1 gramme	= 1000 milligrammes,	mg.
1 gramme	= 100 centigrammes,	cg.
1 gramme	= 10 decigrammes,	dg.
10 grammes	= 1 dekagramme,	Dg.
100 grammes	= 1 hectogramme,	Hg.
1000 grammes	= 1 kilogramme,	Kg.
10000 grammes	= 1 myriagramme,	Mg.
1 quintal	= 100 Kg.	

1 millier or tonneau = 1000 Kg.

The following are approximate values of some of the metric units:

1 metre	= 39·37 inches,	nearly.
1 centimetre	= $\frac{2}{5}$ inch,	"
1 kilometre	= $\frac{5}{8}$ mile,	"
1 hectare	= $2\frac{1}{2}$ acres,	"
1 litre	= $1\frac{3}{4}$ pints,	"
1 stere	= $\frac{1}{4}$ cord,	"
1 kilogramme	= $2\frac{1}{5}$ pounds,	"
1 millier or tonneau	= 2200 pounds,	"

## DEFINITIONS OF ARITHMETICAL TERMS

**Agent.**—A person authorized to transact business for another.

**Annuity.**—A payment made annually, or at other periodic intervals, for a fixed term of years or during the lifetime of some person.

**Assets.**—The property of a debtor available for the satisfaction of his creditors.

**Average.**—The average of several quantities is that quantity which, when repeated as many times as the number of the quantities, will give an aggregate equal to the aggregate of these several quantities.

**Bank Discount.**—The amount deducted from the value (at maturity) of a note.

**Bill of Exchange.**—A written order directing some person (or a bank) to pay to some other person a stated sum of money.

**Bonds.**—Written or printed contracts, made under seal, promising to pay a certain sum of money at a specified time. They are issued for money borrowed by Governments, Cities, Towns and other Corporations. They usually bear interest which is paid at certain stated times.

**Broker.**—A man who buys or sells stocks for others.

**Brokerage.**—Payment made to a broker for buying or selling stocks.

**Chord of a Circle.**—A straight line joining two points on the circumference.

**Commission.**—An allowance or payment made to a commission merchant or agent for the transaction of business; it is usually calculated as a percentage on the money actually involved in the transaction.

**Compound Interest.**—Interest not only on the principal, but on the interest added to the principal when it becomes due.

Banks usually add the interest to the principal every six months.

**Cone.**—A solid whose base is a circle and whose curved surface tapers uniformly to a point called the vertex.

**Consignee.**—The person (or firm) to whom goods are sent.

**Consignment.**—A shipment of goods sent to a commission merchant to be sold.

**Consignor.**—The person (or firm) who sends goods to another.

**Consols.**—An abbreviation for **Consolidated Annuities**, the funded debt of Great Britain.

**Corporation.**—An association of persons for the transaction of business; its legal rights are in many respects the same as those of a single person.

**Coupon.**—An interest certificate attached to a bond; to be detached and presented for payment when the interest is due.

**Course of Exchange.**—The rate connecting the units of value of two countries, according to which bills of exchange for the time being are negotiated.

**Creditor.**—A person (or firm) to whom money is due.

**Cube Root.**—One of the three equal factors of a number.

**Customs.**—Taxes or duties imposed on imported or exported goods.

An *ad valorem* duty is calculated as a certain percentage of the value of the goods as shown in the invoice.

A *specific* duty is assessed upon the number, weight or measure of the goods, without regard to value.

**Days of Grace.**—Three Days of Grace are allowed by law after a note is nominally due (that is, after the expiration of the time named in the note for its payment,) before it is legally due.

**Debenture.**—A deed or contract by which certain property is charged with the repayment of money lent, together with interest at a fixed rate. Debentures usually have interest coupons attached.

**Debtor.**—A person (or firm) owing money to another.

**Deferred Annuity.**—An annuity that does not begin until some future time.

**Discount.**—An abatement or reduction from a stated price or value.

**Dividend.**—The sum divided among the stockholders of a company as the profits of the business.

**Draft.**—An order directing some person (or a bank) to pay a specified sum of money to some other person.

**Duty.**—See Customs.

**Endowment Policy.**—An insurance policy that secures to the person insured a certain sum of money at a specified time, or to his heirs if he die before that time.

**Equated Time.**—The time at which several sums of money due at different times may be paid.

**Equation of Payments.**—The process of finding the equated time.

**Evolution.**—The process of finding roots of numbers.

**Exchange.**—The system by which persons living in different countries, or persons living in distant parts of the same country, discharge their debts to each other.

**Exponent.**—See Power.

**Factor.**—A number which will divide a given number without a remainder, is called a factor of that number.

**Frustum.**—The portion of a cone or pyramid included between its base and a plane parallel to the base.

**Instalment.**—A payment made annually, or at some other stated period.

**Insurance.**—A guarantee of payment of a specified sum of money in the event of the loss of property by fire, shipwreck, etc., or loss of life.

**Interest.**—The sum paid for the use of money.

**Liabilities.**—Debts.

**Net Proceeds.**—The amount that remains of the money received for property after paying the expenses incurred in disposing of it.

**Net Price.**—The price of goods after the trade discount has been deducted from the marked price.

**Note.**—A written promise to pay a specified sum of money at a stated time and place.

**Parallelepiped.**—A solid having six faces, each face being a parallelogram. If the faces are all rectangles, the solid is called a rectangular parallelepiped.

**Partial Payment.**—Part payment of a note, bond or other obligation.

**Par of Exchange.**—The rate connecting the unit of value of one country with the unit of value of another country, only the intrinsic values of the units being considered.

**Partnership.**—The association of two or more persons with joint capital, for the carrying on of some particular business.

**Par Value.**—The par value of a stock certificate or bond is the original value or the amount stated in the certificate.

**Percentage.**—The result obtained by taking a certain number of hundredths of a given quantity.

**Perpetual Annuity, or Perpetuity.**—An annuity that continues forever.

**Plinth.**—A rectangular parallelepiped having two opposite faces square.

**Policy.**—The written agreement or contract guaranteeing insurance.

**Poll Tax.**—A tax levied equally on all taxable persons.

**Polygon.**—A plane figure bounded by more than four straight lines.

**Polyhedron.**—A solid bounded by planes.

**Power.**—When a product consists of the same factor repeated any number of times it is called a **Power** of that factor. The number of times that the factor is repeated is called the **Exponent** or **Index** of the Power.

**Preference Stock.**—That part of the stock of a company on which a certain percentage must be paid before any dividend can be declared on the ordinary stock.

**Premium.**—The sum paid for insurance. Also the excess of market value above par value.

**Present Worth.**—The present value of a sum of money due at some future time.

**Principal.**—The sum of money for the use of which interest is paid.

**Prism.**—A polyhedron of which two opposite faces are parallel rectilineal figures, connected by plane faces at right angles to their parallel faces.

**Pyramid.**—A polyhedron of which one face is a rectilineal figure and the other faces are triangles having a common vertex.

**Rectangle.**—A right-angled parallelogram.

**Right-Angled Triangle.**—A triangle, one of the angles of which is a right angle. The side opposite the right angle is called the **Hypotenuse**.

**Root.**—A root of a number is one of the equal factors of the number.

**Sector of a Circle.**—A plane figure bounded by two radii of a circle and the part of the circumference intercepted by these radii.

**Share.**—One of the equal parts into which the capital of a company is divided.

**Stocks.**—The shares of companies and the bonds of governments and corporations.

**Sphere.**—A solid bounded by a surface, every point of which is equally distant from a certain point within, called the centre.

**Square Root.**—One of the two equal factors of a number is called the square root of the number.

**Trade Discount.**—A deduction from the regular or marked price of an article.

**Trapezium.**—A four-sided plane figure having two sides parallel.

**True Discount.**—The difference between the present worth of a debt and the whole debt; hence true discount is equivalent to the interest on the present worth of the debt.

**Wedge.**—A triangular prism.

# Answers

## PAGE

- 2.—1. Two hundreds; five tens; six units. 2. It shows that there are no tens in the number; if it were omitted the 3 would represent tens not hundreds. 3. One thousand two hundred and thirty-four; twelve hundred and thirty-four. 4. 99; 999. 5. 345; 543. 7. The first one to the left, the first to the right.
- 3.—8. Five-dollar bills; 25c. pieces; 1c. pieces. 9. 599993; 500003. 10. 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004. 11. 12; 9. 12. \$500. 13. 40. 14. 2 yards. 15. B is 27 times A; 27. 16. 100 bushels, 400 pecks, 800 gallons, 3200 quarts, etc.
- 7.—1. 77033. 2. \$813060.20. 3. 351. 4. 5328. 5. 9929790. 6. The latter by 1627758. 7. 9000. 8. \$325, \$436, \$239.
- 8.—9. 2055, 2270. 10. (1) 45684—19479; (2) 784562—87183. 11. 2001. 12. 15 feet. 13. 8425. 14. 133; 57; 19. 15. 228. 16. \$313.65, \$6.15. 17. 9562. 18. 5. 19. 5. 20. 613. 21. 1392. 22. 6862. 23. 25.
- 9.—24. \$2858.75. 25. 341. 26. 14887838; 468528; 1270843. 27. (1)  $823 \times 15$ ; (2)  $1524 \times 32$ . 28. 50c. 29. \$5.23. 30. 39. 31. 356 32. 20c.
- 10.—33. \$43.35. 34. (1) 3425; (2) 34568. 35. 35. 36. 20941592. 37. 85650. 38. 4071375. 39. 3837; 19185. 40. 21729. 41. 12045. 42. 91, 32 rem. 43. 281484; 3096324; 31244724.

## PAGE

- 15.—1. 8; 46; 6; 42. 2. 19; 23; 33. 3. 360; 1440. 4. 21645; 218929. 5. 720720. 6. (1) 2. 3. 5. 7. 11. 13. (2)  $2^1$ .  $3^2$ . 7. 13. 7. 16, including 1. 8. 2, 3, 7. 9. 3080. 10. 29.
- 16.—11. 3000. 12. 84 sec. 13. 360. 14. 337. 15. 14364. 16. 32 or 96. 17. 119 or 595. 18. 330. 19. 12832. 20. 40 grains; 144 lb. avoird.
- 20.—1.  $3\frac{1}{4}$ . 2. 9. 3.  $5\frac{7}{8}$ . 4.  $8\frac{6}{7}$ . 5.  $27\frac{1}{4}$ . 6. 9. 7. 32. 8.  $80\frac{42}{43}$ . 9.  $45\frac{540}{963}$ . 10.  $11\frac{1}{909}$ . 11.  $100\frac{1}{101}$ . 12. 1001. 1.  $\frac{7}{2}$ . 2.  $\frac{21}{4}$ . 3.  $\frac{55}{8}$ . 4.  $\frac{143}{12}$ . 5.  $\frac{124}{7}$ . 6.  $\frac{579}{10}$ . 7.  $\frac{211}{15}$ . 8.  $\frac{455}{19}$ . 9.  $\frac{31285}{9}$ . 10.  $\frac{22306}{23}$ . 11.  $\frac{15923}{6789}$ . 12.  $\frac{332928}{577}$ .
- 21.—1.  $\frac{24}{7}$ ,  $\frac{36}{7}$ ,  $\frac{48}{7}$ ,  $\frac{60}{7}$ ,  $\frac{96}{7}$ . 2.  $\frac{153}{19}$ ,  $\frac{221}{19}$ ,  $\frac{391}{19}$ ,  $\frac{799}{19}$ . 3.  $\frac{4900}{157}$ ,  $\frac{9300}{157}$ ,  $\frac{25600}{157}$ .
- 22.—1.  $\frac{1}{6}$ . 2.  $\frac{48}{77}$ . 3.  $\frac{56}{135}$ . 4.  $\frac{1683}{200000000}$ . 5.  $\frac{1}{720}$ . 6.  $\frac{16000}{969}$ . 7.  $\frac{32}{9}$ . 8.  $\frac{3618274}{2040}$ . 9.  $\frac{2288}{81}$ . 10.  $\frac{21989}{2997000}$ . 1.  $\frac{5}{14}$ ,  $\frac{5}{21}$ ,  $\frac{5}{28}$ ,  $\frac{5}{56}$ . 2.  $\frac{7}{144}$ ,  $\frac{7}{180}$ . 3.  $\frac{148}{46875}$ .
- 23.—1.  $\frac{9}{12}$ ,  $\frac{15}{20}$ ,  $\frac{36}{48}$ ,  $\frac{75}{100}$ . 2.  $\frac{20}{28}$ ,  $\frac{21}{28}$ . 3.  $\frac{49}{63}$ ,  $\frac{54}{63}$ . 4.  $\frac{40}{60}$ ,  $\frac{45}{60}$ ,  $\frac{48}{60}$ . 5.  $\frac{20}{24}$ ,  $\frac{21}{24}$ . 6.  $\frac{10}{18}$ ,  $\frac{3}{18}$ ,  $\frac{12}{18}$ . 7.  $\frac{30}{36}$ ,  $\frac{28}{36}$ ,  $\frac{33}{36}$ ,  $\frac{18}{36}$ ,  $\frac{10}{36}$ . 1.  $\frac{1}{3}$ . 2.  $\frac{3}{4}$ . 3.  $\frac{3}{4}$ . 4.  $\frac{2}{3}$ . 5.  $\frac{3}{5}$ . 6.  $\frac{2}{7}$ . 7.  $\frac{2}{3}$ . 8.  $\frac{3}{4}$ . 9.  $\frac{1}{3}$ . 10.  $\frac{1}{3}$ . 11.  $\frac{5}{8}$ . 12.  $\frac{7}{9}$ .
- 24.—13.  $\frac{7}{9}$ . 14.  $\frac{11}{13}$ . 15.  $\frac{1547}{1881}$ . 16.  $\frac{47957}{54549}$ . 1.  $\frac{1}{5}$ . 2.  $\frac{1}{2}$ . 3.  $\frac{3}{7}$ . 4. 1. 5.  $\frac{3}{20}$ . 6.  $\frac{33}{7}$ . 7.  $\frac{107}{8}$ . 8.  $\frac{915}{98}$ . 9.  $\frac{113}{123}$ . 10.  $\frac{1}{4}$ .
- 25.—1.  $\frac{54}{6}$ ,  $\frac{63}{7}$ ,  $\frac{90}{10}$ ,  $\frac{900}{100}$ . 2.  $\frac{546}{14}$ ,  $\frac{741}{19}$ ,  $\frac{897}{23}$ .
- 27.—1.  $\frac{70}{90}$ ,  $\frac{48}{90}$ ,  $\frac{15}{90}$ . 2.  $\frac{40}{60}$ ,  $\frac{45}{60}$ ,  $\frac{48}{60}$ ,  $\frac{50}{60}$ . 3.  $\frac{2352}{2520}$ ,  $\frac{980}{2520}$ ,  $\frac{2415}{2520}$ ,  $\frac{2280}{2520}$ . 4.  $\frac{1000}{10000}$ ,  $\frac{100}{10000}$ ,  $\frac{10}{10000}$ ,  $\frac{1}{10000}$ . 5.  $\frac{49}{273}$ ,  $\frac{208}{273}$ ,  $\frac{87}{273}$ . 6.  $\frac{255}{357}$ ,  $\frac{90}{357}$ ,  $\frac{7}{357}$ ,  $\frac{238}{357}$ ,  $\frac{17}{357}$ . 7.  $\frac{690}{2760}$ ,  $\frac{300}{2760}$ ,  $\frac{2185}{2760}$ ,  $\frac{552}{2760}$ ,  $\frac{2720}{2760}$ . 8.  $\frac{3}{8}$ ,  $\frac{55}{8}$ ,  $\frac{32}{8}$ . 9.  $\frac{28}{65}$ ,  $\frac{720}{65}$ ,  $\frac{325}{65}$ . 10.  $\frac{143}{15015}$ ,  $\frac{39}{15015}$ ,  $\frac{35}{15015}$ . 1.  $\frac{7}{8}$ . 2.  $\frac{7}{9}$ .

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28.—3.  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ . 4.  $\frac{17}{24}$ ,  $\frac{2}{3}$ ,  $\frac{7}{12}$ . 5.  $\frac{8}{7}$ ,  $\frac{1}{12}$ ,  $\frac{3}{40}$ . 6.  $\frac{7}{12}$ ,  $\frac{11}{21}$ ,  $\frac{29}{60}$ ,  $\frac{5}{16}$ . 7.  $2\frac{5}{11}$ ,  $\frac{22}{9}$ ,  $2\frac{3}{8}$ ,  $\frac{16}{7}$ . 8.  $\frac{15}{14}$ ,  $\frac{6}{11}$ ,  $\frac{13}{27}$ ,  $\frac{9}{25}$ . 9.  $\frac{23}{24}$ ,  $\frac{22}{23}$ ,  $\frac{19}{20}$ ,  $\frac{18}{19}$ .

29.—1.  $\frac{6}{7}$ ,  $\frac{7}{8}$ . 2.  $\frac{16}{17}$ ,  $\frac{18}{19}$ . 3.  $\frac{17}{15}$ ,  $\frac{21}{19}$ ,  $\frac{25}{23}$ . 4.  $\frac{7}{17}$ ,  $\frac{12}{21}$ ,  $\frac{16}{25}$ . 5.  $\frac{11}{13}$ ,  $\frac{6}{7}$ . 6.  $\frac{4}{5}$ ,  $\frac{5}{6}$ ,  $\frac{7}{8}$ . 7.  $\frac{3}{97}$ ,  $\frac{5}{161}$ ,  $\frac{7}{225}$ ,  $\frac{11}{353}$ . 8.  $\frac{3}{19}$ ,  $\frac{7}{44}$ ,  $\frac{5}{31}$ .

30.—1.  $\frac{14}{5}$ . 2.  $\frac{12}{13}$ . 3. 1. 4.  $1\frac{11}{35}$ . 5.  $1\frac{17}{24}$ . 6.  $1\frac{53}{72}$ . 7.  $2\frac{13}{60}$ . 8.  $2\frac{23}{60}$ . 9.  $3\frac{807}{840}$ . 10.  $\frac{14432}{64467}$ . 11.  $13\frac{1}{2}$ . 12.  $13\frac{3}{4}$ . 13.  $22\frac{2321}{2520}$ . 14.  $15\frac{47}{924}$ . 15. 1.

31.—1.  $\frac{3}{5}$ . 2.  $\frac{2}{17}$ . 3.  $\frac{1}{8}$ . 4.  $\frac{1}{6}$ . 5.  $\frac{2}{143}$ . 6.  $\frac{1}{96}$ . 7.  $\frac{1}{198}$ . 8.  $23\frac{38}{3}$ . 9.  $\frac{19}{420}$ . 10.  $\frac{241}{420}$ . 11.  $\frac{1}{19}$ . 12.  $\frac{1}{56}$ . 13.  $\frac{1}{812}$ . 14.  $\frac{1}{7}$ . 15.  $\frac{29}{140}$ . 16.  $2\frac{2}{15}$ . 17.  $17\frac{1}{99}$ . 18.  $\frac{53}{80}$ .

32.—1.  $\frac{8}{9}$ . 2.  $\frac{10}{21}$ . 3.  $\frac{8}{105}$ . 4.  $\frac{343}{720}$ . 5.  $\frac{3}{7}$ . 6. 1. 7.  $2\frac{7}{34}$ . 8.  $162\frac{1}{2}$ . 9.  $\frac{35}{72}$ . 10.  $\frac{9}{400}$ . 11.  $\frac{2}{3}$ . 12.  $\frac{1}{7}$ . 13.  $\frac{15}{56}$ . 14.  $3\frac{111}{74}$ . 15.  $\frac{47}{84}$ .

33.—1.  $\frac{4}{5}$ . 2. 2. 3.  $1\frac{5}{91}$ . 4.  $\frac{7}{9}$ . 5.  $\frac{18}{19}$ . 6.  $\frac{19}{100}$ . 7.  $\frac{1}{11}$ . 8. 99. 9.  $\frac{25}{32}$ . 10. 9. 11.  $\frac{99}{140}$ . 12.  $1\frac{17}{28}$ . 13.  $1\frac{1}{35}$ .

35.—1.  $1\frac{11}{45}$ . 2.  $5\frac{7}{16}$ . 3.  $8\frac{2}{5}$ . 4.  $\frac{7}{30}$ . 5.  $\frac{1}{5}$ . 6.  $\frac{8}{21}$ . 7.  $1\frac{1}{7}$ . 8.  $\frac{45}{301}$ . 9.  $1\frac{109}{210}$ . 10.  $7\frac{7}{19}$ . 11.  $3\frac{475}{1008}$ .

40.—1. Seven, and six-tenths. 2. Thirty-nine, and three-tenths. 3. Four, and eighty-nine hundredths. 4. Seven hundred and sixty-two. 5. Seven hundred and sixty-two thousandths. 6. Seven hundred and sixty-two, and seven hundred and sixty-two thousandths. 7. One thousand two hundred and thirty-four, and five thousand six hundred and seventy-eight ten-thousandths. 8. One hundred and twenty-three, and forty-five thousand six hundred and seventy-eight

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- hundred-thousandths. 9. Two thousand four hundred, and thirty-six ten-thousandths. 10. Two thousand four hundred and thirty-six ten-thousandths. 11. Six ten-thousandths. 12. Six millionths. 13. Six billionths. 14. 76·89. 15. 14·003. 16. 100·0003. 17. ·0103. 18. 30070·001083.
- 42.—1. 3525·9774. 2. 61·23737. 3. 2503·61876243.  
4. 95·222524. 5. 4·3345. 6. 6·5754.
- 44.—1. 2·4493944. 2. 5·477082. 3. 44·395031413.  
4. 57·8687277. 5. 7·7123875248. 6. ·26484624.  
7 12025·21200096. 8. ·00000072. 9. ·0000001.  
10. 1.
- 45.—1. 1·2, ·12, ·012. 2. ·4, ·04, ·16, 12·8, 64.
- 46.—3. 8·1, ·81, ·9, 2·7, 218·7, ·0003. 4. ·015625, 78·125, 625, ·000025, ·625. 5. ·00121, ·161051, ·014641, ·01331. 6. 1·368, ·05472, 2·736, ·1368, ·0684.  
7. 2799360, 77760, 129600, 216000. 8. ·5, ·2, ·125, ·04, ·015625, ·008, ·00390625, ·0016. 9. ·47712.  
10. ·43241. 11. ·69315. 12. ·31830.  
1.  $\frac{31}{32}$ . 2.  $\frac{13}{16}$ . 3.  $\frac{4}{625}$ . 4.  $\frac{1}{3125}$ .
47. 1. ·25, ·5, ·75, ·125, ·375, ·625, ·875. 2. ·1875, ·3125, ·4375, ·5625, ·6875, ·8125, ·9375. 3. ·2, ·4, ·6, ·8. 4. ·96, ·992, ·9984. 5. ·96875, ·984375, ·9921875, ·00390625. 6. ·142857, ·285714, ·428571, ·571428, ·714285, ·857142. 7. ·918918, ·675675, ·531531, ·438438, ·386386. 8. ·333333, ·666666, ·166666, ·833333, ·777777, ·727272, ·692307.
- 52.—1. ·947368421052631578, ·631578947368421052, ·5882352941176470, ·3529411764705882, ·615384, ·3913043478260869565217, ·380952.

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2.  $\dot{.6086956521739130434782}$ ,  
 $\dot{.4482758620689655172413793103}$ ,  $\dot{.967741935483870}$ ,  
 $\dot{.972}$ ,  $\dot{.02439}$ ,  $\dot{.3584905660377}$ .  
 3.  $\dot{.879120}$ ,  $\dot{.269841}$ ,  $\dot{.987012}$ ,  $\dot{.386100}$ ,  $\dot{.518}$ ,  $\dot{.000351}$ ,  
 $\dot{.999000}$ .

54. 1.  $\frac{2}{3}$ ,  $\frac{8}{11}$ ,  $\frac{1}{11}$ ,  $\frac{43}{111}$ ,  $\frac{669}{1111}$ ,  $\frac{5}{7}$ . 2.  $\frac{36}{185}$ ,  $\frac{1}{10}$ ,  $\frac{1}{48}$ ,  $\frac{4}{35}$ .  
 3.  $2\frac{1}{3}$ ,  $3\frac{1}{6}$ ,  $14\frac{37}{990}$ ,  $7\frac{51}{350}$ .

55.—1. 3.9283. 2. 1.6854. 3. 40.90586. 4. 2621.108687.

57.—1.  $\dot{.6359723}$ , 2.  $\dot{1.715399467874}$ . 3.  $\dot{1.67660503}$ .  
 4.  $\dot{.402392970}$ .

60.—1. 148.862. 2. 1.338. 3. 1.1918. 4. .435.  
 5. 70.5839.

63.—1. 29.995. 2. 856.967. 3. 1.413. 4. 29.956.  
 5. 13.593. 6. 1.407. 7. 262.856.  
 1.  $\dot{.0202}$ ,  $\dot{.2025}$ ,  $\dot{202.522}$ . 2. 138.280 or 138.281.  
 3.  $1\frac{42929}{874791}$ . 4.  $\dot{.0285714}$ ,  $\dot{.047619}$ . 5.  $\dot{.8398}$ ,  $\dot{.8397}$ .  
 6. 16.8476190. 7.  $\dot{.00015625}$ . 8.  $\dot{.4047}$  or  $\dot{.4048}$ .  
 9.  $\frac{1}{47}$ . 10.  $\frac{38}{50}$ . 11.  $\frac{479}{1170}$ . 12.  $\frac{2721}{1001}$ . 13. .52.

64.—14. 3.141592. 15.  $10\frac{4}{7}$ . 16.  $\frac{17}{18\frac{15}{19}}$ . 17. .3183.

18. 409. 19.  $\frac{28}{37}$ ,  $\frac{155}{1147}$  and  $\frac{47}{98}$  will circulate with  
 periods of not more than 36, 36, 97 digits respec-  
 tively. 20.  $909\frac{1}{11}$  oz. 21. 6. 22. 100. 23. \$2.10,  
 \$2.73, \$.63, \$.7. 24. 10.1873. 25. 14. 26. \$1365.  
 27.  $83\frac{1}{3}$ c.

65.—28. \$320. 29.  $19\frac{1}{2}$  yd. 30. 11c. 31.  $\frac{1}{11}$ . 32. \$1200,  
 \$900. 33. 1c. 34. \$612, \$720, \$864, \$1080. 35. 35.  
 36. \$1.31 nearly. 37.  $\frac{1}{10}$ ,  $\frac{1}{15}$ ,  $\frac{1}{20}$ ;  $4\frac{8}{15}$ . 38. \$22.50,  
 \$37.50. 39. 9 years.

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- 66.—40. \$12937.50. 41. \$3.56 nearly. 42. \$3000, \$4500, \$6750, \$9750. 43. \$4076.163. 44. \$1323, \$1260, \$1200. 45. 474 lb. 46. 4.30 p.m. 47.  $19\frac{6}{8}\frac{1}{1}$  gal. 48.  $\frac{2}{5}\frac{8}{8}\frac{7}{7}$ ,  $6\frac{2}{3}$ . 49. .142857. 50.  $62\frac{1}{2}$ c.
- 67.—51. \$175, \$105. 52. 12, 30. 53. \$21.50, \$21. 54. 1900, 1850. 55.  $521\frac{1}{2}\frac{7}{8}$  lb. 56. 30.98. 57. 45c.
69. 1. 198. 2. 311760. 3. 525600. 4. 5451776000. 5. 199. 6. 24000. 7. £10 8s. 4d. 8. 10. 9. Saturday.
- 70.—10. 795. 11.  $312\frac{1}{2}$ . 12. \$81. 13. 2175. 14. 1497. 15. 3041. 16. 88 yr. 5 mo. 10 da. 17. 47. 18.  $\frac{2}{5}$ . 19. Mar. 21st, 1871. 20. 44. 21. 8 min. 25 sec. 22. 2 mi. 1430 yd. 23. 175. 24. 171.6. 25. 84.
- 71.—26. 70. 27.  $2\frac{4}{7}\frac{6}{7}$  oz. 28. 14 da. 7 h. 11 m. 17 sec. 29. 16s.  $9\frac{1}{3}$ d. 30. 146097. 31. 36524. 1. 10, 100, 1. 2. 1, 100, 10000. 3. 1200, 15000, 21000000. 4. 1234567.89, 123456.789, 123.456789. 5. 856000, 5632, 12468000. 6. 1635639.87. 7. 554151.1. 8. 1152.48768 Km. 9. 91.441. 10.  $18\frac{1}{3}$ .
- 72.—11. 50. 12. 179.07. 13. 749.301. 14. 4 m., 6 m. 15. 13 hr. 32 m. 30 sec. 16. 900. 17. 952, 400. 18. 12, 120000, 120. 19. 1017.32. 20. 9.9. 21. 204.2415 Ha. 22. .1. 23. 1500000. 24. 39.6. 25. 1000000. 26. 1. 27. 100. 28. 2500. 29.  $909\frac{1}{11}$  Kg.
- 73.—30: 1000, 1000. 31. 1 Kg. 32. 1000. 33. 1 cu. dm. 34.  $1\frac{2}{7}\frac{3}{7}$  Kg. 35. 1000. 36. 1000000. 37. 1000000. 38. 10000. 39. 13.5. 40. 46 : 25. 41. 907.21. 42.  $138\frac{8}{9}$ . 43. 20.116. 44. 101.17. 45. 368.529. 46. 1609.3. 47. 39.6. 48. 1000.

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74. 1.  $2^2=4$ . 2. 9. 3. 100. 4. 289. 5. 400. 6.  $10^4=10000$ .  
 7. 90000. 8.  $(.1)^2=.01$ . 9. .16. 10. .0036.  
 11. .055225. 12.  $\frac{1}{4}$ . 13.  $\frac{1}{4^2}$ . 14.  $\frac{9}{100}$ . 15.  $5^3=125$ .  
 16. 1000. 17. 1296. 18. .001. 19. .00000016.  
 20.  $7\frac{9}{32}$ . 21.  $7\frac{58}{81}$ .  
 1. 16, 25, 529. 2. .09, .0016, .000256, .268324.
- 75.—3.  $\frac{1}{4}$ ,  $\frac{9}{16}$ ,  $1\frac{21}{16}$ . 4. .0470, .0021, 21.3721. 5. 100,  
 10000, 1000000. 6. 343, 2744, 4330747. 7. .001,  
 .000008, .000000000001. 8.  $\frac{8}{27}$ ,  $\frac{729}{2187}$ ,  $\frac{2197}{1000}$ .  
 9. .33310, .00001. 10. 432, 36125, 144000.  
 11. 2401, 243, 128. 12. 1.4989, 2.3965.
- 76.—1. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 72, 60,  
 90, 140, 480, 800, 9000. 2. 1, 2, 3, 4, 5, 6, 7, 8, 9,  
 10, 14, 24, 45, 20, 50, 80, 126, 270, 200, 900. 3. 6,  
 8, 10, 24, 27, 32, 36, 42, 80, 33, 240, 800, 560, 280,  
 105. 4. 2, 3, 8, 10, 60, 30, 12, 21, 11, 110, 300,  
 35. 5. 2, 3, 5, 4, 6, 20, 21. 6.  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{3}{8}$ ,  $\frac{7}{10}$ ,  $\frac{9}{100}$ ,  
 $\frac{8}{27}$ ,  $\frac{10}{27}$ ,  $\frac{8}{1000}$ , .1, .2, .3, .8, 2.4, 3.2, 4.2, 10.5, 1.05,  $\frac{8}{27}$ .  
 7.  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{4}{5}$ ,  $\frac{7}{10}$ ,  $\frac{10}{9}$ ,  $\frac{4}{21}$ ,  $\frac{35}{16}$ , .1, .2, .4, 1.2, 1.1, 2.1, 3.5,  
 .01, .12, .35.
- 77.—2. 25. 3. 150. 4. 900. 6. 1225. 7. 325. 11. 5.  
 12. 5. 13. 0. 14. 1225.
- 78.—15. 3, 5, 35. 19. 5. 21. 35. 22. 25, 0; 25, 6; 31,  
 0; 31, 9.
- 79.—1. 9. 2. 10. 3. 100. 4. 100. 8. 8516.
- 80.—17. 754. 19. 917. 20. 829, 389, 213, 295.  
 1. 1, .01, .0001, .000001. 2. 1, .1, .01, .001. 6. .3,  
 .07, .009, .002, 1.1, 1.2, 7.1, 5.1, 82.2946, 2.13,  
 17.2522, 18.8281, 51.4732, .6324.

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81.—1. 21, 32, 72. 2.  $3\frac{1}{11}$ ,  $6\frac{3}{29}$ . 3.  $\frac{1}{6}$ ,  $2\frac{1}{6}$ . 4.  $\frac{4}{9}$ ,  $\frac{2}{3}$ ,  $\frac{8}{2}$ .  
 5. 5.1715. 6. .7071. 7.  $3\sqrt{2}$ , .778. 8. 57, 15.  
 9. 76. 10. 3.968. 11. 10919.

82.—12. 9, 3. 15. 335, 336. 16. 75c. 17. 63. 18. 142450.  
 19. 6084. 20. 121, 363. 21. 36, 42. 22. 8, 16, 24.

83.—2. 3. 3. 1. 6. 3. 7. 2. 9. 6. 11. 3. 12. 3. 13. 3.  
 14. 9.

84.—15. 1. 16. 1. 18. 9.  
 3. 1 ft. 4. 4. 6. 40. 7. 10. 8. 10 sq. ft. 9.  $38\frac{1}{2}$   
 sq. rods. 10.  $8\frac{1}{2}\frac{5}{6}$  sq. yd. 11. 6 sq. yd. 6 sq. ft.  
 40 sq. in.

85.—1. 13 ft. 2. 41 ft. 3. 85 ft. 4. 17 yd. 5. 27.784 in.

86.—6. 4 ft. 11.933 in. 7. 8.171 ft. 8. 6 ft. 9. 40 ft.  
 10. 112 ft. 11. 12.649 yd. 12. 3 yd. 10.253 in.  
 13. 4.529 ft. 9. 6 sq. ft.

87.—10. 3.72 sq. yd. 11. 72.727 sq. in.

88.—1.  $1\frac{1}{3}$ ,  $11\frac{1}{3}$ ,  $4\frac{8}{3}$ . 2.  $1\frac{40}{41}$ ,  $39\frac{1}{41}$ ,  $8\frac{3}{41}$ . 3.  $3\frac{8}{85}$ ,  $166\frac{2}{85}$ ,  
 $25\frac{5}{85}$ . 4.  $\frac{1}{2}$ ,  $7\frac{1}{2}$ , 4.974. 5.  $5\frac{3}{98}$  in.,  $43\frac{5}{98}$  in., 71.797  
 in. 6. 30, 180, 2184, 19.899, 1759.035.

90.—1.  $12\frac{4}{7}$  ft. 2.  $18\frac{1}{4}$  yd. 3. 14 yd. 2 ft.  $9\frac{3}{7}$  in.  
 4. 23.470 ft.

91.—5.  $2\frac{1}{2}\frac{9}{2}$  ft. 6. 2 ft.  $3\frac{1}{2}\frac{5}{2}$  in. 7. 3.01 yd. 8.  $75\frac{3}{7}$  ft.  
 9.  $27\frac{5}{21}$  yd. 10. 30 yd. 1 ft. 8 in.

93.—22.  $38\frac{1}{2}$  sq. ft. 23.  $50\frac{2}{7}$  sq. ft. 24. 100 sq. yd.  
 $8\frac{2}{7}$  sq. ft. 25. 129.537 sq. ft. 26. 5.941 sq. in.  
 27.  $9\frac{5}{8}$  sq. ft. 28.  $6\frac{9}{92}$  sq. ft.  
 1.  $\frac{1}{12}$ . 2.  $\frac{1}{12}$ . 3. 28 ft. 4.  $51\frac{6}{11}$  sq. ft. 5. 35 sq.  
 ft. 6.  $4\frac{1}{2}$  sq. ft.

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94.—5. 112 sq. in. 6. 84 sq. ft. 7. 66 sq. ft. 8. 6 ft.  
9.  $11\frac{5}{11}$  in. 10. 2 ft.  $7\frac{2}{11}$  in.

95.—6. 21 sq. ft. 7. 21 sq. ft. 8. 54 sq. ft. 9. 242 sq.  
in. 10. 14 in. 11.  $2\frac{5}{22}$  in. 12.  $12\frac{1}{4}$  in. 13. 4 ft.  
14. 5.744 ft. 15. 8.720 in.

96.—16. 5 ft. 17. 11.496 in. 18.  $204\frac{2}{7}$  sq. in. 19. 269.992  
sq. in. 20. 128.192 sq. ft. 21. 117.5 sq. in.

98.—14. 154 sq. ft. 15. 676.063 sq. ft., 1079.134 sq.  
in., 2665.062 sq. ft. 16. 3.102 ft., 7.937 in., 1.761 ft.

100.—20. 60 cu. ft., 65 cu. yd., 29.25 cu. ft., 163.312  
cu. ft. 21. 144 sq. in. 22. 12 in. 23. 8 ft. 24. 4.773  
yd. 25. 64.058 yd.

101.—12. 140 cu. ft. 13. 12 cu. in. 14. 140 cu. ft. 15. 36  
cu. ft. 16. 295.160 cu. ft.

102.—6. 108 cu. ft. 7. 352 cu. in. 8.  $124\frac{8}{11}$  cu. ft. 9. 16  
sq. ft. 10. 2.256 ft. 11. 6.75 in. 12. 1.948 in.  
13. 4.063 ft.

105.—16. 60 cu. ft. 17. 32 cu. ft. 18. 18 sq. in. 19. 4.535  
in. 20.  $46\frac{2}{3}$  cu. ft. 21. 137.673 cu. ft.

106.—6. 81 cu. ft. 7.  $410\frac{2}{3}$  cu. ft. 8.  $38\frac{1}{2}$  sq. in. 9.  $3\frac{1}{2}$  in.  
10. 3.349 yd.

109.—17.  $1437\frac{1}{3}$  cu. in. 18. 268.190 cu. in.

110.—19. 21 in. 20.  $179\frac{2}{3}$  cu. ft.

111.—1. \$1.64. 2. 120 yd. 3. 56 ft. 4. 2000. 5. 216000.

112.—6.  $28\frac{2}{3}$  sq. ft. 7. \$266. 8. 346.106. 9. 40.  
10. 357.071 ft. 11. 60 ft. 12. 18 ft. 13. 1.  
14. 107.158 yd. 15. \$323.326. 16. 4 yd., 20 yd.  
17. \$504. 18. 5 ch. 19. 25899.37.

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- 113.**—20. 8.64 in. **21.** 2.699 ft. **22.** £7 3s. 4d **23.** 30.805 ft. **24.** 364.641 sq. ft. **25.** 334098.48, 334681.89.  
**26.**  $100\frac{4}{7}$  sq. ft. **27.**  $10\frac{2}{7}$  ft. **28.**  $15\frac{5}{9}$  in. **29.**  $22\frac{3}{11}$  ft.  
**30.** 1.68 ft. **31.** .541 yd. **32.** 18.973 ft., 12.649 ft.  
**33.** Square,  $8\frac{6}{7}$  rods. **34.** \$203.20.
- 114.**—35. 2836. **36.** 1.1 8 in. **37.** 243.721. **38.** 2 ac. 46 sq. rods. **39.** 11.2 ft. **40.**  $51\frac{2}{3}$  sq. yd. **41.** 15.512 in. **42.** \$466.66 $\frac{2}{3}$ . **43.** Square, 225 sq. yd.  
**44.** \$158.56. **45.** .0000918. **46.** 24.64 sq. ft.  
**47.** 5.291 in. **48.**  $64\frac{1}{24}$  sq. ft.
- 115.**—49.  $12\frac{1}{32}$ . **50.**  $\frac{169}{512}$ . **51.** 6. **52.**  $221\frac{1}{7}$  sq. in.  
**53.** \$362.50. **54.** 12. **55.**  $15\frac{5}{7}$  in. **56.** 208 ac.  
**57.** 8400 sq. yd. **58.** 65.948 ft. **59.**  $\frac{8}{9}$  hr.  
**60.** 24.589 in. **61.**  $212\frac{1}{7}$ .
- 116.**—62.  $553\frac{1}{7}$ . **63.** \$59.15. **64.** 6.2 ft. **65.**  $2005\frac{1}{3}$  ac.  
**66.** 23 ft. 4 in. **67.** 2710.19 cu. ft. **68.**  $81\frac{9}{11}$  ft.  
**69.** 6995 cu. ft. **70.** 13.75 sq. yd. **71.**  $\frac{7}{8}$ . **72.** 40.737 sq. in. **73.**  $1437\frac{1}{3}$  cu. in.
- 117.**—74. 126 in., 674 sq. in. **75.**  $29\frac{1}{3}$  cu. in. **76.**  $14\frac{1}{7}$  cu. in. **77.** 25. **78.**  $22\frac{1}{2}$ . **79.** 488. **80.** 12.728 lb.  
**81.** 758.556 cu. in. **82.** 792.261 cu. in. **83.**  $44\frac{11}{12}$  cu. in. **84.** 398.353 sq. ft. **85.** .577 in. **86.**  $97\frac{8}{7}$  sq. in.
- 118.**—87. 8 to 27. **88.**  $30\frac{4}{5}$  days. **89.** \$25.099. **90.**  $32\frac{3}{5}$ .  
**91.** 48 in. **92.** 5.744 ft. **93.** 525 sq. ft. **94.** 20.  
**95.** 1325.481 sq. yd. **96.** 189.52 sq. ft. **97.** 4500.  
**98.**  $1257\frac{1}{7}$ . **99.**  $32\frac{3}{4}$  lb. **100.** 2475.
- 119.**—101. 640 tons. **102.**  $235\frac{5}{7}$ . **103.** 14.661 in.  
**104.** 3784.075. **105.**  $11407\frac{11}{7}$  oz. **106.** 44.9.  
**107.**  $\frac{16}{77}$  ft. per hr. **108.**  $5391\frac{27}{8}$ . **109.** 6250.  
**110.** 25.441. **111.** 123.4. **112.** 1720.

## PAGE

- 120.—113. 54. 114. 13. 115. 4·5. 116. 6000. 117. 1.  
 118. 721·1. 119. 90. 120. ·66. 121. 27.  
 122. 1000. 123. 24. 124. 3 m., 12 m., 15 m.  
 125. ·4 m. 126. 1 min. 127. 36 min. 128. 10 m.
- 121.—129. ·331 s. 130. 10 m. 131. ·84. 132. ·375 m.  
 133. 1·155. 134. 364·8.
- 135.—7. 1, 1, 3, 2, 3, 1, 7, 17. 9. 0. 10. It fails when  
 0 or 9 is struck out. 11.  $45^{\circ}$  E.,  $105^{\circ}$  W. 12.  $53^{\circ}$  W.
- 136.—13. 3 hr. 57 min. 56 sec. 14.  $79^{\circ} 24'$  W. 15.  $26^{\circ}$   
 $31'$  W. 16.  $60^{\circ}$  W. 17. 4 hours. 18. 1.20 p.m.  
 19. 5 min. 56 sec. fast, 10 min. 44 sec. fast, 1 min.  
 32 sec. slow. 20. 3 : 4. 21. 36c., 64c. 22. 55.  
 23. \$28·75, \$40. 24. 250, 550, 1150. 25. \$15·65.  
 26. 7, 14, 21, 28.
137. 27. \$1008, \$1296, \$1344. 28. 44, 31. 29. \$165,  
 \$65. 30. 71 : 13. 31. \$10. 32. 5 : 1. 33.  $16\frac{2}{3}$  gal.  
 34. 1 gal. 1 qt.  $\frac{5}{7}$  pt. 35. \$175, \$163. 36. 30, 24.  
 37.  $135\frac{5}{9}$  yd. 38. B,  $8\frac{8}{9}$  yd. 39.  $\frac{2}{3}$  of way round.  
 40. 6 min., starting point.
- 138.—41. 540 miles. 42.  $8\frac{1}{3}$  sec.,  $6\frac{9}{11}$  sec. 43. 2 miles.  
 44.  $4\frac{4}{5}$  miles. 45.  $10\frac{1}{11}$  min. past 2. 46.  $5\frac{5}{11}$  min.  
 past 7. 47.  $21\frac{9}{11}$  min. past 3. 48.  $7\frac{41}{107}$ . 49. 84,  
 42, 21 days. 50. 6 days. 51. 48 min. 52. \$2,  
 $\$1\frac{1}{3}$ , 40c.
- 139.—53.  $18\frac{6}{5}$  days. 54. \$70·50, \$49·50. 55. 90, 180  
 days. 56. 9 : 10. 57. \$7·05, \$6·93. 58.  $5\frac{1}{2}$  days.  
 59. 540. 60. \$8·64. 61. \$22·40. 62. \$128. 63. 2700.  
 64. \$135·13 $\frac{1}{2}$ .
- 140.—65. \$376·20. 66. 65 $\frac{1}{4}$ . 67. \$51·33 $\frac{1}{3}$ . 68.  $42\frac{1}{8}$ .  
 69. \$54·50, or \$53·50 if no waste in border.  
 70. \$10·50. 71. 60. 72. \$7·20. 73. 93 $\frac{5}{8}$ .  
 74. \$47·26 $\frac{3}{4}$ .

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- 141.—1.  $\frac{1}{8}$ ,  $\frac{7}{20}$ ,  $\frac{2}{6}$ ,  $\frac{1}{6}$ ,  $\frac{3}{4}$ ,  $\frac{7}{8}$ ,  $\frac{6}{7}$ ,  $\frac{3}{8}$ , 1,  $\frac{9}{8}$ . 2. 60,  $16\frac{2}{3}$ ,  $26\frac{2}{3}$ ,  $22\frac{1}{2}$ ,  $18\frac{2}{11}$ ,  $187\frac{1}{2}$ , 25,  $36\frac{1}{2}$ ,  $8\frac{3}{4}$ .
- 142.—3. 4, 75, 4. 4, 3, 2. 5. 20,  $16\frac{2}{3}$ , 5,  $5\frac{5}{9}$ ,  $4\frac{1}{2}\frac{6}{11}$ . 6. 216 men, 108 bu. 7. \$4.7753. 8. \$2475000. 9. 420 marks, 57.60 francs. 10.  $6\frac{1}{4}$ . 11.  $2\frac{1}{2}$ . 12.  $12\frac{1}{2}$ . 13. 130. 14. 96. 15.  $1\frac{1}{9}$ ,  $8\frac{8}{9}$  lb. 16. 200. 17. 250. 18. 800. 19. 36.08. 20. 4.
- 143.—21. 500. 22. 96578. 23. \$3360. 24. 35.2. 25.  $42\frac{9}{7}$ . 26. \$8800. 27.  $13\frac{1}{3}$ . 28. 120, 240. 29. 581.4, 579.2, 489.4, 350 lb. 30. 25.9. 31. 75. 32. \$9. 33. 125. 34.  $42\frac{1}{2}$ .
- 144.—35. \$120000. 36. \$2000. 37. 600, 480, 360. 38. 40. 39. 76.8. 40.  $1\frac{9}{21}$ . 41.  $21\frac{1}{8}\frac{9}{8}$ . 42. \$16000. 43. 600. 44.  $1029\frac{7}{17}$ .
- 146.—1. \$432, \$675, \$1966.50, \$787.50, \$458.64. 2. \$150. 3. 15%. 4. \$4.28 $\frac{2}{3}$ . 5. \$16.66 $\frac{2}{3}$ . 6. \$4.57 $\frac{1}{7}$ . 7. \$3.75. 8. 81. 9. 12c. 10. \$6.
- 147.—11. \$10. 12. \$28.12 $\frac{1}{2}$ . 13. 45c. 14. 2% loss. 15.  $3\frac{1}{8}$ . 16.  $1\frac{2}{7}\frac{7}{9}$ . 17. \$4. 18.  $11\frac{1}{8}$ %. 19. Former, \$10. 20. 20%. 21.  $16\frac{1}{9}$ . 22. 28%. 23.  $5\frac{5}{9}$ %. 24. 20%. 25.  $\frac{1}{4}$ %.
- 148.—26. \$4.05. 27. \$2.33 $\frac{1}{3}$ . 28. 50c. 29. \$327.50, \$311.12 $\frac{1}{2}$ , \$294.75. 1. \$264, \$234.30, \$64.05, \$245.35, \$1451.99, \$4055.28.
- 149.—2. 20% gain,  $33\frac{1}{3}$ % loss, 25% loss,  $16\frac{2}{3}$ % gain. 3. \$720, \$560, \$3000, \$7250. 4. \$2.10. 5. \$2887.50. 6. \$10.10 $\frac{1}{10}$ . 7.  $4\frac{1}{10}$ c. 8. 22 $\frac{5}{7}$ c. 9. No. 10. 71 $\frac{1}{2}$ . 11. 50. 12. 50. 13. 15.

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- 150.—14. \$5.75. 15.  $6\frac{1}{4}$ . 16.  $5\frac{5}{8}$ . 17. \$10. 18.  $11\frac{1}{3}$ .  
 19. 1. 20.  $37\frac{1}{10}$  c. 21.  $33\frac{1}{3}$ . 22.  $12\frac{1}{2}$ . 23.  $63\frac{7}{11}$ .  
 24.  $\frac{4}{21}$  gal. 25.  $1\frac{3}{8}$ . 26. 220. 27. \$235.
- 151.—28. 116. 29. \$500. 30.  $\$1129\frac{1}{4}$ . 31. 48c.  
 32. Lost,  $9\frac{7}{10}$  c. 33.  $83\frac{1}{3}$ . 34.  $33\frac{1}{3}$ . 35. \$71 $\frac{2}{3}$ .  
 36.  $42\frac{5}{7}$ . 37. 46·6c. 38. \$200. 39. \$15360.  
 40. \$8·80, no.
- 152.—1. \$54, \$33·60, \$9·60, \$189, \$8·25, \$182·25, \$7·50.  
 2. \$5850. 3.  $66\frac{2}{3}$  c.
- 153.—4. 2%. 5. 4%. 6. 2%. 7. 80. 8.  $\frac{3}{4}$ %. 9. \$16·25.  
 10. \$102. 11. \$25. 12. \$1715·69. 13. \$70.  
 14.  $3\frac{1}{3}$ %. 15. \$945. 16.  $25263\frac{3}{8}$ .
- 154.—17. \$9720. 19. 311963. 20. 100000. 21.  $1\frac{1}{5}$ %.  
 22. 120000 lb.
- 155.—23.  $7847\frac{1}{3}\frac{1}{4}$ . 24. \$54. 25. \$8000. 26. \$5333 $\frac{1}{3}$ .  
 27. \$4060. 28.  $40058\frac{5}{3}\frac{4}{7}$ . 29.  $1\frac{1}{2}$ .
- 156.—31.  $1\frac{1}{2}$ %, 1%. 32. 2%. 33. 2%. 34.  $1\frac{1}{4}$ %.
- 158.—1. \$30, \$112·50, \$27·20, \$246, \$60, \$11·34. 2. \$625,  
 \$2500, \$4450, \$5500. 3.  $1\frac{1}{4}$ %,  $\frac{3}{4}$ %, 55c., 54c.  
 4. \$105. 5. \$56·25. 6. \$160. 7.  $\frac{3}{4}$ %. 8.  $\frac{2}{3}$ %.  
 9.  $\frac{2}{3}\frac{1}{8}$ %. 10.  $1\frac{1}{3}$ %.
- 159.—11. \$2133 $\frac{1}{3}$ . 12.  $\frac{5}{8}$ . 13.  $\frac{3}{4}$ . 14. \$40. 16. \$8000.  
 17. \$4000. 18. \$16000. 19. \$6000. 20.  $1\frac{3}{8}$ %.  
 21.  $1\frac{1}{4}$ %.
- 160.—22. \$312·50, \$400, \$875, \$15151·52, \$24242·42,  
 \$60606·06. 23. \$2000. 24. \$25600. 25.  $\frac{1}{2}\frac{1}{4}$ %.  
 26. \$42256·62. 27. \$10·15. 28. \$4666 $\frac{2}{3}$ . 29. \$759  
 loss. 30. \$3950, \$50. 31. \$32·50.

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- 161.—32.  $1\frac{1}{2}\%$ . 33.  $1\%$ . 34. \$51750. 35. \$6283 $\frac{7}{11}$ ,  
\$4189 $\frac{1}{11}$ . 36. \$11970. 37. \$18312.91.
- 163.—1. \$100. 2. \$37.50. 3. \$240. 4. \$30. 5. \$9375.  
6.  $11\frac{1}{3}$  mills. 7. \$14112. 8.  $16\frac{2}{3}$  mills. 9. \$14.40.  
10. \$16. 11. \$1979. 12. 16 mills. 13. \$6400.
- 164.—14. 15 mills. 15. \$507500. 16. \$4. 17. \$30.  
18. \$14 $\frac{5}{8}$ . 19. \$1900. 20. \$32.37 $\frac{1}{2}$ . 21. \$23.75.  
22. \$780000, \$132600, \$386100, \$179400, \$42900,  
\$19500, \$19500.
- 166.—1. \$120, \$180; \$80, \$125; \$150, \$300; 0, \$32.50;  
\$27, \$42; 0, \$42; \$116.25, \$155; \$102.18, \$119.21;  
\$200, \$300. 2.  $16.03\%$ . 3. \$3.70. 4. \$3.25, \$2.34.  
5. \$300. 6. \$3.78. 7.  $7\frac{1}{2}c$ .
167. 8. \$2.35 nearly. 9. \$2.88. 10. \$20. 11. \$162.73+.  
12. \$76.17+. 13. \$1. 14.  $43\frac{3}{4}\%$ . 15. \$20.  
16.  $10\%$ . 17. 20 doz., \$8. 18. 27 in.
- 168.—19. 3 yd. 20.  $2\frac{1}{2}$  lb.
- 169.—1. \$1250 each. 2. \$1120, \$1680. 3. \$4000.  
4. \$2000. 5. \$20000. 6. \$1200. 7. \$1001.74,  
\$459.13, \$939.13. 8. \$5422.58.
- 170.—9. \$2618.18. 10. 8, 10, 12 mo. 11. \$2000 each.  
12. \$3764.25. 13. \$400, \$800, \$1200, \$2400.  
14. \$6315.79, \$5684.21. 15. \$1333 $\frac{1}{3}$ , \$1666 $\frac{2}{3}$ .  
16. \$8000, \$9000, \$3000.
- 171.—17. \$47.25, \$49, \$80. 18. \$273 $\frac{1}{3}$ , \$133 $\frac{1}{3}$ , \$73 $\frac{1}{3}$ .
- 173.—1. \$1800, \$1250, \$815, \$380, \$1200, \$2000, \$2490,  
\$1397.50, \$1630, \$732.50. 2. \$1470. 3. \$3487.50.

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- 174.—4. \$30, \$240. 5. \$65. 6. \$60. 7.  $3\frac{2}{3}\%$ . 8. 5, \$100; 8, 0; 12, \$22; 26, \$12; 8, \$40; 5, 0; 4, \$4; 7, \$21.75; 6, \$22; 13, \$47.75. 9. 40 shares. 10. 40 shares. 11. \$320. 12. \$87.50. 13. \$118.25. 14.  $5\frac{5}{9}$ . 15.  $5\frac{5}{9}$ . 16.  $8\%$ . 17. 40 shares, \$3010. 18. 140.
- 175.—19.  $110\frac{1}{2}$ . 20. \$352 gain. 21. 1005 sh. 22. 1500 sh. 23.  $78\frac{1}{8}$ . 24. \$480. 25. 84. 26. \$11868.13. 27.  $3\%$ . 28.  $5\%$ . 29. \$600000.
- 176.—30. \$120000. 31. \$327.50. 32.  $20\%$ . 33. 75. 34. \$6500, \$250. 35. 108. 36. \$41400. 37. 80 sh. 38. \$7200, \$10800. 39. 60c. gain.
- 177.—1. (a) \$144. (b) \$131.25.
- 178.—1. (c) \$129.71. (d) \$255.864. (e) \$163.487. (f) \$108.904. (g) \$404.416. (h) £216 1s. 6d. 2. \$106. 3. \$1344. 4. \$60.449. 5. \$180.64. 6. \$1431.493. 7. \$5610. 8. \$4. 9. \$65. 10. 3. 11. 4. 12. 3. 13. 5. 14. 5. 15.  $2\frac{2}{3}$  yr.
- 179.—16.  $16\frac{2}{3}$  yr., \$1159.78. 17. 25 yr. 18.  $33\frac{1}{3}$  yr. 19.  $\frac{3}{25}$ . 20.  $\frac{1}{6}$ . 21. (a)  $\frac{3}{10}$ , (b)  $\frac{27}{200}$ , (c)  $\frac{21}{100}$ , (d) 1, (e)  $\frac{1}{25}$ , (f)  $\frac{1}{50}$ , (g)  $\frac{2}{25}$ . 22. 12. 23. 20. 24. 16,  $18\frac{2}{11}$ , 50 yr. 25. 50,  $33\frac{1}{3}$ , 28 yr. 26. \$3200. 27. \$60. 28. 4.
- 180.—29.  $2\%$ ,  $4\%$ . 30.  $2\frac{1}{2}\%$ ,  $5\%$ . 31.  $4\%$ ,  $3\%$ . 32. \$102. 33. \$525. 34. \$800. 35. \$1300. 36. \$723.938. 37. The latter. 38.  $\$261\frac{1}{11}$ . 39. \$450. 40. \$456. 41. A's offer. 42. \$1389.15.
- 181.—43. \$1170. 44. \$384 at the end of 6 mo. 45.  $4\frac{1}{2}$ . 46.  $1\frac{1}{2}$  yr. 47. 5 yr. 48. \$298.78.

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- 186.—1. (a) Sept. 4th, (b) 92 d., (c) \$12·60, (d) \$987·40.  
 2. (a) Apr. 18th, (b) 76 d., (c) \$16·92, (d) \$1233·08.  
 3. (a) Sept. 26th, (b) 86 d., (c) \$106·32, (d) \$5534·43.  
 4. (a) Mar. 4th, (b) 70 d., (c) \$31·86, (d) \$2737·14.
- 187.—5. (a) Aug. 4th, (b) 61 d., (c) \$3·68, (d) \$271·32.  
 6. (a) Mar. 3rd, (b) 92 d., (c) \$80·66, (d) \$3919·34.  
 7. (a) Nov. 8th, (b) 157 d., (c) \$31·86, (d) \$1202·70.  
 8. (a) Apr. 18th, (b) 63 d., (c) \$24·40, (d) \$2332·10.  
 9. (a) Mar. 3rd, (b) 33 d., (c) \$2·89, (d) \$397·11.  
 10. (a) June 6th, (b) 97 d., (c) \$12·26, (d) \$564·49.  
 11. (a) May 9th, (b) 80 d., (c) \$6·39, (d) \$479·66.
- 188.—12. (a) May 6th, (b) 63 d., (c) \$27·90, (d) \$1992·81.  
 13. (a) Nov. 26th, (b) 86 d., (c) \$80·58, (d) \$4194·33.  
 14. (a) Mar. 3rd, (b) Mar. 3rd, (c) Mar. 2nd, (d) May 3rd, (e) Mar. 3rd. 15.  $\frac{1}{100}$ . 16.  $\frac{5}{100}$ . 17.  $\frac{5}{100}$ .  
 18.  $\frac{1}{100}$ . 19.  $\frac{1}{100}$ . 20. \$365. 21. \$888·50. 22. \$365.
- 189.—23. 6%. 24. 5%. 25. 7%. 26.  $12\frac{1}{2}\%$ . 27. \$365.  
 28. July 7th. 29. Nov. 2nd. 30. \$1199·87.  
 31.  $7\cdot127\%$ . 1. \$35.
- 190.—2. \$465. 3. \$465. 4. \$48·23. 5. \$951·77.  
 6. \$2083·23. 7. \$2104·06. 8. \$60. 9. \$20.  
 10. \$121·64. 11. \$1118·36. 12. \$3942·26.
- 191.—13. \$244·52. 14. \$288·12. 15. \$474·53.  
 16. \$398·74.
- 193.—1. 5. 2. 3. 3. \$3600.
- 194.—4. 40 days. 5. 16 days. 6.  $3\frac{3}{13}$ . 7. 44 days, July 15th. 8. 20 days. 9. 9 mo. 10. 15 days. 11. Apr. 13th. 12. 32 days after debt was due. 13. 120 days. 14. 7 days. 15. 4 days.

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- 195.—16. Sept. 10th. 17. Sept. 2nd. 18. \$923·87.
- 198.—1. \$112·4864. 2. \$1·124864. 3. \$121·55. 4. \$32·33.  
5. \$109·29. 6. \$197·03. 7. \$689·83. 8. \$1201·31.  
9. \$662·45.
- 199.—10. \$9·72. 11. \$4449. 12. \$1025·72. 13. \$1350·61.  
14. \$111·49. 15. \$·113525. 16. \$4328·25. 17. 43923.  
18. \$474·72. 19. \$1·64166. 20. \$1·17416.  
21. \$15400. 22. \$10000. 23. \$729·53.
- 200.—24. 7 + yr. 25. 5. 26. 6·09. 27. 2·95. 28. 5%.  
29. \$10000. 30. Between 28 and 29 years, between  
10 and 11. 31.  $4\frac{1}{2}$ . 32. 5. 33. 12.  
34. \$144, \$109·55.
- 203.—1. \$600. 2. \$769·23. 3. 96·15c. 4. 89c. 5. \$7396·45.  
6. \$23·53. 7. \$48·80. 8. \$14·14, \$97·97, \$93·14,  
\$2852·98, 21·65c. 9. \$1089·30. 10. \$1200·62.
- 204.—11. \$384·82. 12. \$97·572. 13. \$524·42. 14. Latter,  
\$17. 15. \$132. 16. 2. 17. Latter, 5c. 18. Latter.  
19. 5·45. 20. \$85·29. 21. \$1922·75. 22. \$1·80.
- 205.—23. \$1030. 24. \$19 $\frac{1}{11}$ . 25. \$1930·69. 26. \$4037·25.  
27. \$2723·25.
- 212.—1. \$5·23. 2. \$3568·90, \$7629·525, \$6403·59,  
\$2112·05. 3. \$3751·50. 4. \$7481·30. 5. 1% dis-  
count. 6. \$849·15. 7. \$470·15. 8. \$486·66 $\frac{2}{3}$ ,  
\$99·76 $\frac{2}{3}$ , \$11·58, \$373·50, \$42·46, \$29·75. 9. \$2·35.  
10. \$1587·60. 11. \$1070·70. 12. \$1 = 5·22 fr.
- 213.—13. \$349·79. 14. \$290·70. 15. \$5764·50. 16. \$7275.  
17. £500. 18. 9 $\frac{1}{2}$ . 19. 5·16%. 20. £2321 17s. 4d.,  
£276 8s. 3d. 21. \$12279·60. 22. \$342·857.  
23. \$1 = 2·487 fl.

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- 214.—1. \$52.25 loss. 2. \$14.88. 3. \$550, \$371.25.  
 4. \$2826, \$2355. 5. 25. 6. \$144. 7. \$5.25.  
 8. \$133.87½.
- 215.—9. \$6.0172. 10. \$3485. 11. \$180. 12. 11½. 13. \$40.  
 14. \$180, \$240, \$300. 15. \$4383. 16. \$157.50.  
 17. \$270. 18. 209.592 sq. ft. 19. \$4.8545. 20. 360.
- 216.—21. 5½%. 22. \$6979.50. 23. \$90. 24. 28.  
 25. \$16000, \$4320. 26. \$8.50. 27. \$1015, \$1421,  
 \$2233. 28. \$12, \$10. 29. 6¼. 30. \$891. 31. \$65.40.  
 32. 2¼ yr. 33. .0000315.
- 217.—34. 4½. 35. \$2436.66⅔. 36. \$1973684.21. 37. \$75,  
 \$126. 38. 49½. 39. \$3600. 40. \$998.40.  
 41. \$2733.60. 42. 48.6c. 43. \$144, \$180, \$202.50.  
 44. \$3.92. 45. \$1179.
- 218.—46. \$34.046. 47. 26½. 48. ½. 49. 108 yd. 50. 44,  
 9½, 6⅔. 51. 8½⅔ cu. ft. 52. 16⅔. 53. \$22.50. 54. ⅕.  
 55. \$325. 56. \$1720. 57. 25. 58. \$9483.125.
- 219.—59. \$3. 60. \$900. 61. 327, 109. 62. \$2000. 63. 8½⅔.  
 64. 7.6c. 65. 4.8%. 66. 4⅕ sq. yd. 67. \$29.16.  
 68. £4500. 69. 2 to 5. 70. \$18. 71. \$10489.45.
- 220.—72. \$6000. 73. \$340.55. 74. 6¼c. loss. 75. \$500.  
 76. \$1681.355. 77. 43c. 78. 28⅓. 79. \$574.22.  
 80. 75, 60. 81. \$240, \$210. 82. \$225. 83. \$45062.50.
- 221.—84. 3600. 85. 17 mills. 86. 66¼%. 87. \$41.903.  
 88. 11⅓c. 89. \$16293.75. 90. 12½ mills. 91. 858.  
 92. Former. 93. 1⅛%. 94. \$320, 6%. 95. \$730.
- 222.—96. \$1039.54. 97. \$2250. 98. \$935. 99. 20.  
 100. \$1725.22.  
 1. 52⅘. 2. 21, 28, 42. 3. \$3.42, \$1.125, \$2.34,  
 \$1.26, \$4.59, \$1.575, \$14.31.

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- 223.—4. 30, 20 rods. 5. 27000. 6. \$25.50, \$13.50. 7. 5, 7, 9, 13. 8. \$2000, \$3000, \$4000. 9. \$1.1952. 10. 84. 11. 11.2%. 12. \$3 loss. 13. \$4000. 14. 4% loss. 15.  $9\frac{1377}{147}\%$ .
- 224.—16. \$18.42. 17. 18 mills. 18. 5, 7. 19. \$480, \$720, \$900, \$1050. 20.  $12\frac{1}{2}\%$ . 21. \$20. 22. 72514 shares. 23. 25. 24.  $4\frac{296}{2401}$ . 25. 12.
- 225.—26.  $6\frac{1}{3}\frac{8}{7}$  days. 27. \$33.44. 28. \$250. 29. \$875. 30. \$1648. 31.  $\frac{1}{2}\%$ . 32. \$1270. 33. 60480. 34.  $2\frac{1}{2}\%$  loss. 35. \$45.36. 36. \$840, \$700, \$420.
- 226.—37. \$250, 6%. 38. 113.137. 39. B,  $\frac{5}{19}$  yd. 40.  $41\frac{2}{3}$ . 41. \$8.40, \$7, \$6. 42. \$124.368. 43. \$14400. 44. \$7.02. 45. 39. 46. \$3125. 47. \$1980. 48. \$25.92.
- 227.—49.  $16\frac{2}{3}$ . 50.  $11\frac{1}{2}\%$  prem. 51. \$80. 52.  $42\frac{9}{7}$ . 53. 510 lb. 54. 35. 55. B pays A \$16. 56. \$2.67. 57. \$600000. 58. \$78. 59. 5%. 60.  $\frac{9}{44}$ .
- 228.—61. \$2517.50. 62. \$11069.767. 63. \$30, \$29.70. 64. \$2600. 65. \$1714.28 $\frac{4}{7}$ , \$2285.71 $\frac{3}{7}$ . 66. 123.582 cu. in. 67. 30, 20. 68. 5.76 min. 69.  $37\frac{1}{8}$ . 70.  $4\frac{9}{10}$  mo. 71. \$5.25.
- 229.—72.  $5\frac{1}{2}$  yr., \$420. 73. \$1542.85 $\frac{3}{7}$ . 74. \$6.44. 75. \$900, \$400. 76.  $7\frac{1}{5}$  days. 77. \$11.07. 78. 1000000. 79. 90, 20. 80. \$1457.50. 81.  $94\frac{2}{7}$ . 82. \$699.856.
- 230.—83. \$69.33 $\frac{1}{3}$ . 84. \$262.50. 85. \$100 loss. 86.  $2\frac{2}{3}$ c. 87. 1311.566 Kg. 88. \$39.92. 89. \$54.25. \$71.75. 90. \$5600. 91. 196. 92. B,  $2\frac{2}{3}$  yd. 93.  $8\frac{1}{2}$ .

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- 231.—94. \$3128, \$1552. 95. 28, 14. 96. \$1. 97. \$2482·165.  
 98. 1313, 2121. 99.  $\frac{4000}{6828}$  oz. 100. \$2472·94.  
 1. \$30000. 2. \$9·18 $\frac{3}{4}$ . 3. \$400000.
- 232.—4. \$273·40. 5. \$49, \$320. 6. 24 mi. 7. 8%. 8. 21 $\frac{1}{8}$ ,  
 17 $\frac{5}{7}$ , 9. 6c. 10. 5 $\frac{1}{3}$  days. 11. 53·4375c.  
 12. \$7880·299. 13. 5 $\frac{1}{2}$ %.
- 233.—14. 11·49 rods. 15. 33 $\frac{1}{2}$ . 16. 3 $\frac{1}{2}$  min.,  $\frac{1}{2}$  way round.  
 17. \$26666 $\frac{2}{3}$ . 18. \$230, 4 $\frac{1}{2}$  yr. 19. \$6246·675.  
 20. 4 $\frac{1}{2}$ . 21. 75c., 90c. 22. \$8874. 23. \$6000.
- 234.—24. \$9·10, \$9. 25. 113636 $\frac{4}{11}$  oz. 26. 71·8c. 27. 17 $\frac{3}{8}$ .  
 28. \$1016·065. 29. \$15000. 30. \$4048·40.  
 31. \$1462·50. 32.  $\frac{2}{3}$  of the way round. 33. \$504,  
 \$432, \$255.
- 235.—34. 132. 35. \$2241·28. 36. \$240, \$192, \$180.  
 37. 6 $\frac{2}{3}$  loss. 38. 19 min. 36 sec. 39. \$1656. 40. 24,  
 56. 41. 24 $\frac{2}{5}$  loss. 42. \$643·50. 43.  $\frac{2}{5}$ %. 44. 180,  
 135, 108 days.
- 236.—45. \$1120, \$1100. 46. \$3·33 $\frac{1}{3}$ . 47. 26 min. 40 sec.  
 48. \$4900·50. 49. 6%. 50. 24 $\frac{4}{9}$ . 51. 4 $\frac{1}{2}$ , 5 $\frac{1}{4}$  mi.  
 52. 11 $\frac{5}{8}$  mo. 53. \$588·24. 54.  $\frac{32}{248}$ . 55. \$563·297.
- 237.—56. \$17·50. 57. \$3432. 58. \$9107. 59. \$655·36.  
 60. \$3000. 61. 40, 30 mi. 62. \$18 loss. 63. 98  
 shares. 64. 6, 9, 10 mo.
- 238.—65. \$11087·52. 66. \$829·637. 67. 57 $\frac{7}{8}$ . 68. 21·213  
 in. 69. \$3381·75, \$3908·25. 70. 30 $\frac{7}{9}$ c. 71. \$30·80,  
 \$30·40, \$30·10. 72. \$7·36. 73. 100 shares.
- 239.—74. \$100 loss. 75. 26 lb. 76. \$1100. 77. 55 $\frac{1}{2}$ .  
 78. \$116. 79. 50. 80. Latter. 81. \$12400, \$14200.

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- 240.—82. 1, 5 mi. per hr. 83. \$40, \$68.75, \$56.25.  
 84. \$13800. 85.  $8\frac{1}{3}$ c. 86.  $\$9.61\frac{1}{3}$ . 87. 22.4 in.  
 88. \$8, \$14, \$20. 89. \$674. 90. \$119. 91. 8%.
- 241.—92. \$340.75. 93. 105. 94. 9.54%. 95. \$390.  
 96. \$500. 97. \$180, \$210. 98. 3%. 99. \$4.50.  
 100.  $8\frac{1}{2}\frac{6}{5}$  mo.
- 242.—1.  $83\frac{1}{3}$ c. 2. June 11th. 3.  $\frac{20}{51}$  gain. 4. 30720. 5. \$80.  
 6. \$734.40. 7. \$3950, \$50. 8. \$650, \$647.92.  
 9. 818 yd.
- 243.—10. 38. 11. 36. 12. \$5287.87. 13. \$8016.64.  
 14. 40, 60. 15. \$18.40. 16.  $31\frac{1}{4}$ . 17. \$10.50.
- 244.—18.  $4\frac{1}{2}$  mo. 19. \$5970. 20. 136. 21. 265 days, 10.  
 22. \$9000. 23. \$382.12. 24. \$918.02. 25. Aug.  
 16th. 26. 20 to 21. 27. 10%.
- 245.—28.  $\$14\frac{2}{3}$  gain,  $\$51\frac{1}{3}$  loss. 29. 8.9. 30. \$11.20.  
 31. 5%. \$200. 32. \$400000. 33. \$7.20. 34. 50.  
 35. \$451.20, \$12. 36. 81.
- 246.—37. 12675. 38. \$4744.186. 39. \$842.30, \$918.87,  
 \$1598.83. 40. \$117.85 $\frac{5}{7}$ . 41. Dec. 3rd. 42. 22.54%.  
 43. 5.86c. loss. 44.  $\frac{7}{9}\frac{5}{7}$ . 45. \$884.13. 46. \$2.20 $\frac{3}{4}$ .  
 47. \$3045, \$812.
- 247.—48. \$9409.28,  $171\frac{8}{88}\%$ . 49. \$6.72. 50. Crosswise,  
 $4\frac{4}{5}$  sq. yd. 51. \$46.85. 52.  $1\frac{3}{4}$ . 53. \$10000.  
 54. \$96.45. 55. 65. 56.  $1\frac{3}{4}$  ac. 57. 10.2.
- 248.—58. 75c. 59. 6 mo. 60.  $69\frac{7}{8}$ . 61. 6. 62. 25%.  
 63. \$235.20. 64. 40 lb. Troy. 65. \$3463.09. 66. 33.
- 249.—67. 26.56c. 68. 40. 69.  $339\frac{1}{17}$  cwt. 70. 192000.  
 72.  $8\frac{8}{15}\%$ . 73. 5%, \$1260. 74.  $38\frac{1}{2}$  sec. 75. \$29.56.

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- 250.—76. 4th. 77. 30, 36, 45. 78. \$75. 79. \$480, \$600, \$700, \$787.50. 80. \$952.09. 81. \$948.015.  
82.  $20\frac{240}{258}$ . 83. £8750. 84.  $13\frac{1}{9}$  mo. 85. \$2400, \$3000, \$6480.
- 251.—86. 4096. 87. 6 sec. 88. 16.83. 89. 5. 90. \$2 gain. 91. Every  $65\frac{5}{11}$  min. 92. 35, 15. 93. 910, 102, 80 oz. 94. \$173.85.
- 252.—95.  $\frac{1}{4}$ . 96.  $7\frac{1}{2}$  min.,  $\frac{1}{2}$  way round. 97. .43429. 98. 4 min. 99. 12. 100. 2%.  
1. 17, 5 lb. 2. 15.406. 3. \$2.51 $\frac{3}{7}$ .
- 253.—4. 155200. 5. \$6500. 6. \$13383.50. 7.  $\frac{55}{171}$  mi. 8. 165 to 148. 9.  $1909\frac{13}{8}$  oz. 10. 60, 90. 11. \$540, \$370, \$240. 12. Mar. 26th.
- 254.—13. 192.468. 14.  $\frac{3}{8}$  gal. 15.  $10\frac{10}{161}$ . 16. Former, 755 cu. in. 17. 50. 18. 415 to 408. 19. \$15300. 20. \$1200, 4%. 21. \$1038.81. 22. 36 yd.
- 255.—23. \$150,  $6\frac{6}{19}\%$ . 24. 55902 to 24565. 25.  $26\frac{4}{21}$ . 26.  $3\frac{9}{17}$  days. 27. 1 rouble = 4.012 fr. 28.  $7\frac{49}{85}\%$ . 29.  $38\frac{1}{2}$  sq. ft. 30.  $\frac{7}{25}$ . 31. 60c. 32. 2%.
- 256.—33. 10 mo. 34. 20. 35. 5, 1, 1, 3. 36. £250. 37. \$5400, \$8400. 38. 20, 15, 10 ft. 39. \$1455.83. 40. 93. 41. 2 hr.
- 257.—42. \$1.33 $\frac{1}{3}$ . 43.  $17\frac{1}{7}$  min. past 12. 44. 1.942. 45.  $8\frac{4}{7}$ . 46. \$317.33. 47. 178 lb. 48.  $33\frac{1}{3}$ . 49. 12.35 a.m. 50.  $26\frac{1}{2}$ . 51. 20%. 52. 4.060401.
- 258.—53. \$8160. 54. \$3428.57 $\frac{1}{2}$ . 55. \$45. 56. \$914.395. 57. 6, 24. 58. \$950.18, 8.086%. 59. 1 hr.; 5, 0, 2,  $3\frac{7}{11}$  min. 60.  $19\frac{5}{7}$  min. 61. \$863.19.

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- ~~279~~ -62. \$1784.32. 63. A \$75 loss, B \$50 gain, C \$125 gain, D \$100 gain. 64. \$400. 65. 2 yd. 66.  $16\frac{2}{3}\%$ . 67. \$4185. 68.  $45\frac{9}{11}$  hr. 69. 688 sq. in., 510 cu. in. 70. £21 16s. 1d.
260. 71. \$9.112. 72. \$300, \$500. 73. \$674.9184. 74.  $3\frac{5}{11}$  miles. 75. 35c. 76. \$135.60. 77. 2. 78. 4528. 79.  $\frac{5}{17}$  mi. 80. 20%. 81. 29397.58 sq. ft.
- 261.—82. 182 yd. 83. \$142.57. 84.  $\frac{5}{6}$  gal. 85. 1 to 33.67. 86. 29.56. 87. 1090 ft. per sec. 88. \$6.52. 89. \$2400. 90. 7.26 a.m. Oct. 8th. 91.  $270\frac{5}{8}$  cu. ft.
- 262.—92. 6. 93. \$1500, 4%. 94. 29 rods. 95. \$120. 96. \$5385.15. 97. 4 mo. 98. 14.3 min. 99.  $24\frac{10}{11}$ . 100. (a) by 12.56c.







JUN 26 1989

